A Real Options Analysis of Dual Labor Markets and the Single Labor Contract*

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Abstract

We study the optimal hiring and firing decisions of a firm under two different firing costs regulations: 1) Dual labor markets characterized by high firing costs for workers with seniority above a threshold ("permanent workers") and by low costs for "temporary workers". 2) The Single Labor Contract, a policy proposal to make firing costs increasing in seniority at the job. We focus on the option value implied by the regulations and obtain some new results: the optimal firing rule is a constant function of worker’s productivity only for permanent workers. For temporary workers it varies with seniority at the job because the firm tries to keep alive the option to fire at low cost. In the Dual regulation the workers more likely to be fired are those close to become permanent. On the contrary, the Single Contract transfers that maximum firing to the new hires. Thus, fired workers are fired sooner under the Single Contract. However, if both regulations have the same average firing cost for workers who become permanent, temporary workers are less likely to be fired in the Single Contract. Moreover, this new regulation increases hiring and average employment duration. It also reduces turnover among temporary workers, but at the expense of higher turnover among permanent workers who are more often replaced by temporary workers.

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1 Introduction

With aggregate unemployment rates reaching double digits in many countries, labor market reforms are at the center of the economic policy debate. This is especially the case in southern European countries characterized by "dual labor markets". A concept that describes labor regulations with two main types of contracts: on one side, permanent contracts protected with high firing costs; on the other side, temporary contracts with low firing costs that must be upgraded to permanent when worker’s seniority at the job reaches a certain threshold.\(^1\) These countries are among those with higher youth unemployment rates (in 2010 about 25\% in France and Italy, more than 40\% in Spain), and at least half of their young workers have a temporary contract (Scarpetta et al. 2010).

Among the different policy proposals, one seems especially popular: unifying the Dual Labor regulations into a Single Contract that would have firing costs increasing in seniority at the job.\(^2\) For example, Nicolas Sarkozy endorsed the idea during the French 2007 presidential election (Cheron 2007); in Spain it is in the electoral program of one of the major political parties (Expansion 2011); in Portugal implementing a version of it was imposed by the EU in the 2011 rescue package (Bentolila 2011).

In this paper we compare the Single Contract and the Dual Labor regulation in a partial equilibrium model that explicitly takes into account the option value implied by the different firing cost regulations. We study the behavior of a firm which can be either active or idle. Active firms employ a worker and make stochastic profits which can be positive or negative. They can fire their worker at any time and become idle by paying a firing cost.\(^3\) If the firm is idle it does not employ any worker and its profits are zero. An idle firm can hire a worker by paying a hiring cost and become active (we assume no matching frictions and perfectly elastic labor supply, i.e. "workers are waiting at the gate").

We model the Dual regulation assuming that the firing cost is a constant if firing happens before worker’s seniority reaches a threshold \(T\), and a higher constant if firing happens after \(T\). For the Single Contract we assume that firing costs start at some positive level and continuously increase with worker’s seniority until seniority reaches \(T\). After this threshold the firing cost is the same constant level than for permanent workers in the Dual Labor.

Under both regulations, to fire before \(T\) is similar to an American option that gives the

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\(^1\)For example, before the 2010 reforms, in Spain firing costs for temporary contracts (job seniority smaller than 3 years) were the wage amount of 8 days of work per year of job seniority. Meanwhile for permanent contracts the costs were the wage amount of 45 days per year of job seniority. The 2010 reform reduced the firing costs to 12 for temporary workers and to 33 days for permanent.

\(^2\)There are several proposals of Single Contract that differ in their details but share this common element: Blanchard and Tirole (2003) and Cahuc and Kramarz (2004) for France, Boeri and Garibaldi (2007) and Ichino et al. (2009) for Italy, and a manifesto signed by 100 academic economists (Andrés et al. 2009) for Spain.

\(^3\)We do not consider fixed term contracts that imply zero firing cost at the expiration of the contract.
right of firing at low costs. This implies that when the option is alive the optimal firing rule is not only a function of worker’s productivity, but also of both the time to expiration of the option, and of the cost of exercising it. Firms with permanent workers do not have that option, thus their firing behavior only depends on the productivity of the worker. The Dual regulation and the Single Contract differ on the timing of the costs of exercising the option, what changes radically the firm’s behavior. In the Dual regulation the workers more likely to be fired are those close to become permanent because the firm tries to keep alive the option to fire at low cost. On the contrary, in the Single Contract the option does not have much value for those workers because their firing costs are close to those of permanent workers. In the Single Contract the maximum firing happens with new hires because firms anticipate that the option loses value as worker’s seniority increases. Thus, fired workers are fired sooner under the Single Contract.

We also show that if the regulations share the same average firing cost at \( T \), and the same protection for permanent workers, then the Single Contract increases hiring and reduces turnover among temporary workers, but at the expense of higher turnover among permanent workers who are more often replaced by temporary workers. These results happen because for any duration strictly shorter than \( T \) the Single Contract has lower average and cumulative firing costs. Thus, higher incentives to both hiring and firing. Overall, the Single Contract generates a higher average time employed.

We did comparative statics on the main parameters of the model to check the robustness of the previous results, and to assess the sensitivity of the two regulations. We noticed that when firms become more impatient (higher discount factor) the Single Contract generates more firing of temporary workers than the Dual because the anticipation of future costs plays a higher role in the Single Contract. And for high levels of risk aversion the Single Contract provides less incentives to fire, especially transitory workers.

Our paper is related to two literatures:

1) The paper uses techniques from the literature of investment under uncertainty (Dixit and Pindyck 1994 is an early survey, Cetin and Zapatero 2010, Hugonnier and Morellec 2007 or Miao and Wang 2007, are, among others, recent examples). Bertola and Bentolila (1990) is closely related. They also study a continuous time partial equilibrium labor demand model. However, their firing and hiring costs are linear and do not imply any option value.

2) By the questions studied, our paper complements the search and matching literature that has studied Dual Labor markets (for example, Bentolila et al. 2010, Cahuc and Postel-Vinay 2002, Costain et al. 2011, Dolado et al. 2007 or Sala et al. 2010) and, less intensively,

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4 An American put option is a financial contract in which the buyer of the option has the right, but not the obligation, to sell an agreed financial instrument, to the seller of the option at any time during the life of the option for a certain price.

5 Our results are qualitative. Our model is too stylized for a full quantitative analysis.
the Single Contract (Costain et al. 2011, Garcia-Perez 2009, Garcia-Perez and Osuna 2011). Our main contribution is to show that the option value implicit in the firing regulations makes the optimal firing of temporary workers a function of seniority at the job. Thus, the firing rule is not a constant productivity level because the firm tries to keep alive the option to fire at low cost.

The paper proceeds as follows. Sections 2 describes the model and Section 3 the solution method. Section 4 discusses the results. Section 5 performs comparative statics. Section 6 concludes. Proofs and details of the solution method are in an online Appendix.

2 Model

We analyze an infinitely-lived firm in a continuous-time setting. The firm can be in any of two states: 1) It can be active, employing a worker and receiving a stochastic stream of profits net of wage costs \( y_t \); or 2) it can be idle, have no employee and receive zero net profits. Profits can take either positive or negative values as they evolve as an arithmetic Brownian motion:

\[
dy_t = \mu dt + \sigma dB_t
\]

where \( \mu \) is the expected profit growth (in levels) and \( \sigma \) is the profit growth volatility. Both \( \mu \) and \( \sigma \) are constant.

An active firm can fire the worker at any time but it must pay a firing cost \( q(\tau) \) that depends on how long the worker has been employed in the firm (\( \tau \)). We focus on two cost functions:

i) The Dual Labor market, where the cost of firing a worker is a step function with two levels: if the fired worker has seniority smaller than a threshold \( T \) then the firm has to pay cost \( q \). If the worker has seniority larger than \( T \) then the firing cost is higher (\( \bar{q} \))

\[
q(\tau_t) = \begin{cases} \bar{q} & \text{if } \tau_t \geq T \\ q & \text{if } \tau_t < T \end{cases} \quad \text{with } \bar{q} > q > 0
\]

ii) The Single Contract, where firing costs start at some positive level (\( q_0 \)) and increase linearly with slope \( \mu_q \) as the worker remains employed. Once seniority attains a threshold \( T_S \) the firing cost becomes constant

\[
q(\tau_t) = \begin{cases} q^S(\tau_t) = q_0 + \mu_q \tau_t & \text{if } \tau_t \leq T_S \\ \bar{q}^S = q_0 + \mu_q T_S & \text{if } \tau_t > T_S \end{cases}
\]

\[\mu_q > 0, q_0 > 0\]
If the firm fires its worker it switches to the idle state where net profits are zero. Idle firms monitor potential profits \( (y_t) \) and can hire a worker at any time by paying a hiring cost \( (c) \). If they do so they start producing at the next instant. Thus, the first profit received by an idle firm that hires a worker at \( t \) is \( y_{t+\varepsilon} \), for infinitesimal \( \varepsilon \).

We assume that the firm has subjective discount rate \( \delta \) and it is risk averse. We follow the recent financial literature on firm’s capital structure (Bhamra et al. 2010 or Chen 2010, among others) and assume an exogenous stochastic discount factor unaffected by the firm’s firing/hiring policy. The firm maximizes its value by discounting cash-flows with the stochastic discount factor implied by CARA utility over potential profits:

\[
    u(y_t) = -\frac{1}{\gamma} \exp (-\gamma y_t),
\]

where \( \gamma \) is the coefficient of absolute risk aversion. This is equivalent to the problem of a risk-neutral firm whose discount rate is

\[
    r = \delta + \mu\gamma - \frac{\sigma^2\gamma^2}{2},
\]

and whose risk-adjusted expected profit variation is \(^6\)

\[
    \mu^* = \mu - \gamma\sigma^2.
\]

Since the firm can decide at each time whether to fire or not, profits before firing and hiring costs \( (\pi_t) \) can be written as

\[
    \pi_t = I_t y_t
\]

where \( I_t \) is an indicator function that takes the value one if the firm has a worker at time \( t \),

\(^6\)These parameters are obtained by decomposing the stochastic discount factor

\[
    e^{-\delta t} \frac{u'(y_t)}{u'(y_0)} = e^{-\delta t - \gamma(y_t - y_0)} = e^{-\delta t - \gamma(\mu t + \sigma B_t)} = Z_{0,t} H_{0,t}
\]

into the time discount factor \( Z_{0,t} = e^{-r^* t} \), with \( r^* = \delta + \mu\gamma - \frac{1}{2} \gamma^2 \sigma^2 \), and the risk-neutral density process \( H_{0,t} = e^{-\frac{1}{2} \kappa^2 t - \kappa B_t} \), with market price of risk \( \kappa = \gamma\sigma \). The Radon-Nikodym theorem and the Girsanov theorem imply that

\[
    E \left[ \int_0^\infty Z_{0,t} H_{0,t} \pi_t dt \right] = E^* \left[ \int_0^\infty Z_{0,t} \pi_t dt \right]
\]

where \( E^* \) denotes expectation with respect to the probability measure under which \( B_t^* = B_t + \kappa t \) is a standard Brownian motion (the risk-neutral probability measure). Substituting \( B_t = B_t^* - \kappa t \) into the dynamics of profits \( \pi_t \) and \( y_t \), we obtain the risk-neutral firm’s value.
and zero otherwise. Firing means \( dI_t = -1 \), while hiring implies \( dI_t = 1 \). Thus \( \pi_t \) evolves as:

\[
d\pi_t = I_t[\mu dt + \sigma dB_t] + y_t dI_t
\]  

(8)

The firm’s problem is to decide the optimal times at which to hire (if the firm is idle) or to fire (if it is active) to maximize its expectation of cumulative discounted cash-flows. We denote by \( \eta_i \) the time at which the firm takes those decisions.\(^7\) And use the indicator function \( I_{\eta_i} \) that takes the value one when the firing/hiring decision is taken. Then, the problem of the firm under the risk neutral measure is

\[
V = \max_{\eta_i} E \left[ \int_0^\infty e^{-rt} \pi_t dt \right] - \sum_{i=1}^\infty E \left[ e^{-r\eta_i} I_{\eta_i} q(\tau_{\eta_i}) + (1 - I_{\eta_i})c \right]
\]

s.t. \( d\pi_t = I_t[\mu^* dt + \sigma dB_t] + y_t dI_t \)

s.t. \( dy_t = \mu^* dt + \sigma dB_t \)

s.t. \( q(\tau_i) = \begin{cases} \text{as in (3) for the Single Contract} \\ \text{as in (2) for the Dual Labor market} \end{cases} \)

3 Solving the model

Both cost functions (2) and (3) imply that the value of the firm’s option to fire depends on time, because firing is cheaper if it is done before the employment reaches \( T \) or \( T_S \). To capture this feature of the option value we will solve the model using a randomizing approximation method proposed by Carr (1998) to price American put options with finite maturity. The idea is to convert the problem into one of an infinite-maturity option with a stochastic termination time.

To describe the method let’s assume that \( T = T_S \) and denote it by \( T_S \). Carr (1998) method partitions the employment time threshold \( T_S \) into \( n \) subintervals and it assumes that \( T_S \) is not a deterministic time but a stochastic time denoted by \( \tilde{T} \). The random variable \( \tilde{T} \) has mean \( T_S \), and variance \( Var(\tilde{T}) \) that converges to zero as \( n \to \infty \). Thus, the deterministic case can be approximated with any accuracy by the stochastic case by increasing \( n \).

We assume that the employment time \( (\tau) \) starts in the first time interval and switches randomly to the next one when it receives a shock distributed as a continuous time Poisson process with hazard rate \( n/T_S \). Thus, the average time expected in the first interval is \( T_S n \) and the variance \((T_S n)^2\). The shocks at different intervals are \( i.i.d \). Thus, the average time to have received \( n \) shocks is \( E(\tilde{T}) = T_S \), and the variance \( Var(\tilde{T}) = \frac{(T_S n)^2}{n} \), which converges to zero.

\(^7\)Thus \( i = 1, \ldots, \infty \). If the firm starts in the idle state, the firm is hiring when \( i \) is odd, and it is firing when \( i \) is even.
as \( n \to \infty \).

We denote by \( u \) a state variable that captures how many shocks have happened, or, in other words, in which interval is the employment time. We can write the firing cost \( q(\tau, u) \) as a function of \( u \) since \( q(\tau, u) \) gets into the flat shape of \( \tau_t > T_S \) only after \( n \) shocks.

There are \( n + 1 \) intervals (the first \( n \) before \( T_S \), plus the one after \( T_S \) at which firing costs are constant). Thus, for example, if \( n = 2 \) then \( u = 0, 1 \) or \( 2 \). The variable \( u_t \) changes over time depending on the shocks, it evolves as a continuous-time markov chain with intensity \( \frac{n}{T_S} \). For example, when \( n = 2 \) its intensity matrix is

\[
\begin{bmatrix}
-\frac{2}{T_S} & \frac{2}{T_S} & 0 \\
0 & -\frac{2}{T_S} & \frac{2}{T_S} \\
0 & 0 & 0
\end{bmatrix}
\]

with the third state being an absorbing state.

We denote by \( V(I_t = 1, y_t, q_t, u_t) \) the value function of a firm employing a worker \((I_t = 1)\), receiving profits \( y_t \), facing firing cost function \( q_t \) which depends on the employment duration, and on interval \( u_t \). This firm must decide an optimal time \( \eta \) to fire. This optimal time can be infinite. If it fires, the firm will get the discounted continuation value of an idle firm \( V(I_t = 0, y_\eta, q_0, u_\eta = 0) \). Hence the active firm’s value is

\[
V(1, y_t, q_t, u_t) = \max_\eta E \left[ \int_\tau^\eta e^{-r_s} y_s ds - e^{-r_\eta} q_\eta + e^{-r_\eta} V(0, y_\eta, q_0, 0) \right]
\]  

(10)

The first term is expected cumulative discounted profits until the time of firing. The second term captures the firing costs of firing a worker of duration \( \eta \). The third term is the continuation value.

The optimal \( \eta \) can be expressed as a minimum profit level that triggers firing once attained. We call this profit level the firing boundary, denoted as \( y(q, u) \), which depends on costs \( q \) (hence seniority at the job), and the state variable \( u \) which determines whether costs have switched to constant. For profit values above the boundary the firm prefers to keep the worker. For profits below the boundary the worker is fired and the firm goes idle. Firing occurs the first time the profit value \( y \) reaches the boundary.

When the firm is idle profits are zero, but it can hire at any time \( \eta \). Its value function is

\[
V(0, y_t, q_0, 0) = \max_\eta E \left[ e^{-r_\eta} V(1, y_\eta, q_0, 0) - e^{-r_\eta} c \right]
\]

(11)

The first term is the discounted value upon hiring at time \( \eta \) and becoming an active firm. The second term captures the hiring costs discounted from the hiring time to the present.
There is a critical level of potential profits \( \bar{y} \) that motivates the firm to hire, we call this the hiring boundary. It separates an inactivity region where low profits discourage the firm from hiring, from an activity region, where high profits induce the firm to hire. The hiring boundary depends on hiring costs, the evolution of the profits process, and on firing costs of the firm which just hired.

An online Appendix characterize the firing and hiring boundaries for both regulations and explain our numerical solution. Next section discusses their patterns.

### 4 Theoretical predictions

In this Section we analyze the qualitative predictions of the model. Our model is too stylized for a full quantitative analysis. Given the lack of closed form solutions we solve numerically a somewhat plausible parameterization. We checked that the patterns that we discuss are robust to different parameterizations. Moreover, in Section 5 we study how changes in the parameters affect the results.

#### 4.1 Parameterization

Concerning the dynamics of profits (equation 1), we set the deterministic expected profit increase \( \mu \) to 0.05 and the volatility \( \sigma \) to 0.14. If we measure profits in units of $100 millions this corresponds to a firm experiencing $5 million of expected annual profit increase, with a standard deviation of $14 millions.\(^8\) Concerning the preference parameters, we set the coefficient of absolute risk aversion \( \gamma \) to 3, and the subjective discount rate \( \delta \) to 0.15. Section 5 does comparative statics on these parameters.

Concerning the firing costs, to focus on the differences between regulations due to different shapes of firing costs instead of different levels, we study the case when both regulations give the same protection to permanent workers

\[
\bar{q} = \bar{q}^S
\]

and this maximum protection is attained at the same seniority level

\[
T_S = T = T
\]

\(^8\)This volatility of earnings variation seems conservative for many industries. For example, in the auto sector, between 1947 and 2007, the average annual variation of real before tax profits was -389 millions (in U.S. dollars of 2005), while the standard deviation was much higher, $7584 millions (Bureau of Economic Analysis, NIPA Tables 6.17 A,B,C,D).
Moreover, we assume that both regulations imply the same average firing cost for workers whose seniority is $T$, that is

$$q = \frac{1}{T} \int_0^T q^S(\tau) d\tau$$  \hspace{1cm} (14)$$

As we will discuss below, assumption (14) highlights an important feature of the Single Contract. Even if it is designed to have the same average cost as the Dual for workers that become permanent, its cumulative and average costs are necessarily lower for workers hired before $T$

$$\frac{1}{j} \int_0^j q^d d\tau > \frac{1}{j} \int_0^j q^S(\tau) d\tau \hspace{1cm} \forall \ j < T$$  \hspace{1cm} (15)$$

Panel A of Figure 1, which plots the benchmark firing cost regulations, shows assumptions (12) – (15).

Concerning the firing cost parameters, we set them as multiples of the daily wage, which we assume to be 0.05. This implies a monthly wage of around $1500 for a worker generating an expected annual revenue of $6800 to the firm, if we assume a profit rate of 25% of revenues, and wage costs of 2/3 of revenues (a rough approximation to the labor share in National Income). We have set $q_0 = 45$ wage days, and $T = 3$ given that one period in the model is one year. We set $q_0$, $\mu_q$ and $q$ in order to meet assumptions (12) – (15) with a non-negative $q_0$. The hiring cost (c) does not play an important role in the results, we set it to half of the smallest firing cost (the initial cost of the single contract). Table 1 summarizes the benchmark parameterization.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & Value & Unit \\
\hline
$\mu_q$ & 0.05 & \\
$q_0$ & 45 & Wage days \\
$q$ & $\frac{(6.8+\mu) \div 0.25 \times \frac{2}{3}}{365}$ & Daily wage
\hline
\end{tabular}
\caption{Benchmark Parameterization}
\end{table}

4.2 Results

An active firm fires its worker when the profit level crosses the firing boundary from above. Hence, a higher firing boundary implies a higher incentive to fire. An idle firm hires a worker when the profit level crosses the hiring boundary from below. A lower hiring boundary implies a higher incentive to hire.

Panel B of Figure 1 reports the optimal firing and hiring boundaries under both types of regulations for the benchmark parameterization of Table 1. The regulations imply very different firing patterns, and also different hiring boundaries. First we discuss each regulation separately, then we compare them:

a) The Dual Labor: for $\tau \in [0, T]$ the firing boundary is increasing in seniority at the job,
as seniority approaches $T$ the firm demands more profits to keep the worker employed. Thus, most of the incentive to fire is concentrated at $T$. A pattern that is consistent with the empirical evidence and explained by the option value implicit in the Dual Labor. Firms like to have the option to fire at low cost, and they keep it alive by firing before $T$. Once the worker reaches $T$ the option disappears.

The slope of the firing boundary before $T$ is increasing in the gap in firing costs $(\bar{q} - q)$, and in how close seniority is of $T$. The first effect can be seen in Panel A of Figure 2, which plots the boundary for a lower value of $q$ while keeping $\bar{q}$ constant. The higher the labor protection of the permanent worker relative to the temporary, the higher the value of keeping alive the option to fire at low cost. Moreover, a larger $(\bar{q} - q)$ implies more hiring and more firing around $T$ (the hiring and firing boundaries are closer). This higher turnover is a "churning effect", once temporary workers get close to $T$ they are fired and (soon) replaced by new hires. The firm incurs firing and hiring cost to keep alive the option to fire cheap.

Panel B of Figure 1 also shows that the firing boundary for permanent workers is flat and lower than for temporary workers. It is flat because now there is no option value, firing costs are constant. It is lower because permanent workers are protected by higher firing costs.

b) The Single Contract: the maximum of the firing boundary is at the start of employment ($\tau = 0$) and the firing boundary decreases in seniority. Two reasons explain these patterns: 1) At $\tau = 0$ firing costs are the cheapest. And lower firing costs encourage more firing. 2) Firing costs are increasing (up to $T$) creating an incentive to fire before costs become more expensive. The expected cost increase is maximal at $\tau = 0$, and it decreases progressively to zero as costs are closer to the maximum cost, i.e. as seniority gets closer to $T$. After $T$ the firing boundary is flat and at its lowest level because costs are constant and at their maximum level.

Panel B of Figure 2 shows how the slope of the firing boundary depends on the slope of cost increase $\mu_q$ and on how close seniority is to $T$. It plots firing boundaries for larger $Ts$ and smaller slopes $\mu_q$ of firing costs, while keeping unaltered the firing cost after $T$. We can see that both the intercept and the average slope of the firing boundary decrease as $T$ becomes larger. The slower the transition to the highest firing costs the smaller the anticipation effect, and smaller the incentive to fire. The higher $\mu_q$, the higher the initial incentive to anticipate firing and the faster the boundary decays as employment time goes by.
From the previous discussion we can draw two conclusions from comparing both regulations:

i) Relative to the Dual Labor, the Single Contract transfers most of the incentive to fire from the workers with seniority close to $T$ to those just hired. The extent of this reshaping depends on the rate of cost increase $\mu_q$ in the Single Contract. Figure 3 plots a consequence of this reshaping: the average seniority of fired workers is lower in the Single Contract. This happens for both workers fired before (Panel A) and after $T$ (Panel B). As it is intuitive, workers that started at a higher profit level have on average been employed more time when fired (it took more time for profits to cross the firing boundary).

Insert Figure 3 about here

ii) If the regulations share the same average firing cost at $T$ (condition 14) and the same protection for permanent workers (condition 12), then the Single Contract generates more incentive to hire (lower hiring boundary) and higher turnover among permanent workers (the firing boundary for permanent worker is higher and its distance from the hiring boundary is smaller). Figure 4 confirms these results. Panel A shows that an unemployed worker has a higher probability of being hired under the Single Contract. Panel B shows that for different levels of firm profitability the Single Contract has a slightly higher probability of firing a permanent worker. Panel C shows that the Single Contract has lower probability of firing a transitory worker except for workers starting in very bad profit conditions. These results follow from condition (15), for any duration strictly shorter than $T$ the Single Contract has lower average and cumulative firing costs. Thus, higher incentives to hire and fire. Figure 5 confirms that this is the explanation. Its Panel A proposes a cost structure violating condition (15). And its Panel B shows that for this new cost structure the firing boundary of permanent workers is not anymore higher in the Single Contract. Moreover, now the Dual has lower hiring boundary.

Insert Figure 4 about here
Insert Figure 5 about here

Thus, an important message from Figures 3 and 4 is that during the temporary phase there is less chance of being fired in the Single Contract, but if the worker is fired it happens before than in the Dual Labor, when most of the firing happens at $T$. Overall, the higher likelihood of hiring and lower likelihood of firing transitory workers in the Single Contract generate a higher average time employed. As it is shown in Panel D of Figure 4.
5 Comparative Statics

In this Section we do two things: on one side to check the robustness of the results discussed in Section 4.2. On the other, to assess how changes in the parameters affect firm’s firing behavior. We start with the subjective time-discount factor ($\delta$). Panels A and B of Figure 6 plot the firing boundary as a function of $\delta$ at three different seniority levels ($\tau = 6$ is a permanent worker, $\tau = 2.5$ is worker close to become permanent, $\tau = 0.5$ is a worker hired recently). Two effects are at play. On one hand more impatient firms fire earlier, because they are less willing to tradeoff present losses for future profits. On the other hand, high $\delta$ implies that firing costs today are more expensive relative to future profits, hence an incentive to postpone firing. For $\tau = 0.5$ and $\tau = 2.5$ the first effect dominates and the boundary is monotonically increasing in $\delta$ for both regulations. However, for the workers with higher costs ($\tau = 6$) when $\delta$ is high enough the second effect dominates and more impatient firms fire later. Panel C plots the difference between the firing boundaries of the Dual and the Single as a function of $\delta$ for the same three seniority levels. The Single Contract is more sensitive than the Dual to changes in discount rates at the beginning of the employment relation. Higher $\delta$ makes the Single Contract to generate much more firing of temporary workers than the Dual. This is a consequence of condition (15). Firms anticipate the average cost increase and when they are more impatient they ask for higher profits to keep the worker. The closer seniority is of $T$ the smaller the anticipated cost increase, what favors the Single Contract.

Figure 6 plots the firing boundary for different values of expected risk neutral profit variation ($\mu^*$). Intuitively, in both regulations there is less firing when firms expect higher profits. When the deterministic drift is higher any bad profit shock will be more transitory. The shapes of the boundaries are not affected by $\mu^*$. And Panel C shows that both regulations seem to react similarly to changes in this parameter.

Figure 7 plots the firing boundary for different values of expected risk neutral profit variation ($\mu^*$). Intuitively, in both regulations there is less firing when firms expect higher profits. When the deterministic drift is higher any bad profit shock will be more transitory. The shapes of the boundaries are not affected by $\mu^*$. And Panel C shows that both regulations seem to react similarly to changes in this parameter.

Figure 8 plots the firing boundary for different values of risk neutral profit volatility ($\sigma^*$). Panels A and B show that the shapes of the boundaries are not affected by $\sigma^*$.

An increase of $\sigma^*$ implies two opposite effects: 1) As in any standard option, given that payoffs are asymmetric (exercise in good times, wait in bad times) an increase of the risk-neutral
volatility enhances the value of the option to fire and delays firing. 2) Firing before $T$ is a special option, it is the option to fire at low cost. To keep this option alive the firm cannot let the employment duration last more than $T$. Thus, when higher volatility encourages the firm to keep this option alive, the firm fires sooner. Effect 1) dominates for our parameterization and in Panels A and B, for both regulations, higher $\sigma^*$ reduces firing. But Panel C, shows that effect 2) is there, and it is important when comparing both regulations. Panel C plots the Dual Labor when the cost of firing a permanent worker (the cost gap $\overline{q} - q$) in the Dual Labor is infinite, what makes the option to fire at low cost very valuable. We can see that for new hires effect 1) is still prominent, but close to $T$ an increase of volatility induces the firm to fire earlier. This is effect 2) in play, more volatile firms fire sooner to keep alive the option to fire cheap. Thus, the effects of $\sigma^*$ on both regulations depend crucially on the seniority of the worker.

Figure 9 plots the firing boundary for different values of the risk aversion coefficient $\gamma$ at two different seniority levels ($\tau = 6$ is a permanent worker, $\tau = 0.5$ is a worker hired recently).

Panels A and B show that for both seniority levels, both regulations display a non-monotonic pattern of the firing boundary with respect to an increase in risk aversion. This is explained by equations (5) and (6). Higher risk-aversion lowers $\mu^*$ via equation (6) and, initially, increases the discount rate $r$ of equation (5). As in Figures 6 and 7, both effects push for early firing. However, further increases of $\gamma$ reduce $r$ and induce the firm to fire less. Panel C reports the difference between the firing boundaries of the Dual Labor and the Single Contract. It shows that for high levels of risk aversion the Single Contract provides less incentives to fire, especially transitory workers.

### 6 Conclusions

In this paper we use a real options model to study firing and hiring under two different regulations: the Dual Labor market and the Single Contract. We focus on the option value implied by these regulations. We show that it implies that for temporary workers the optimal firing rule is a function of their seniority because the firm tries to keep alive the option to fire at low cost. Relative to Dual regulations, the Single Contract transfers most of the incentive to fire from workers close to become permanent to new hires. Thus, fired workers are fired sooner under the Single Contract. However, if both regulations have the same average firing cost for workers who become permanent, temporary workers are less likely to be fired in the Single Contract. Moreover, the Single Contract increases hiring and average employment duration. It
also reduces turnover among temporary workers, but at the expense of higher turnover among permanent workers who are more often replaced by temporary workers. These results may be especially important in a model where workers can invest in human capital. Or in a model with search costs or other frictions related to turnover.

Our model focused on qualitative patterns and abstracted from several dimensions important in quantitative work, for example, differentials in wage and productivity between workers of different seniority, or general equilibrium effects.
References


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Figures and Tables

Parameters of Profit Process

\[
\begin{align*}
\mu &= 0.05, \quad \sigma = 0.14 \\
\end{align*}
\]

Preference Parameters

\[
\begin{align*}
\delta &= 0.15, \quad \gamma = 3 \\
\end{align*}
\]

Firing and Hiring Cost Parameters

\[
\begin{align*}
T_S = T = 3, \quad \mu_q &= (40/3) \times 0.05, \quad q_0 = 5 \times 0.05, \quad q &= 25 \times 0.05, \quad \bar{q} = 45 \times 0.05, \quad c = 0.5q_0,
\end{align*}
\]

Table 1. Benchmark Parameterization. This table shows the parameters used to numerically solve the model and construct the figures of Section 4.

![Panel A: Firing Cost Regulations](image)

![Panel B: Optimal Firing and Hiring Boundaries](image)

Figure 1. Firing Costs Regulations and Optimal Hiring and Firing Boundaries.
Panel A plots the benchmark firing cost regulations of Table 1. Panel B plots the results for those parameterizations.
Figure 2. Comparative Statics on the Cost of Firing a Transitory Worker. Panel A shows the effects of increasing the gap in firing costs in the Dual regulation. Panel B plots the Single Contract for different slopes of firing costs that satisfy Conditions 12 and 14.

Figure 3. Expected Employment Times for Fired Workers. Panel A plots the average seniority of a worker fired before becoming permanent as a function of the starting profit level. Panel B plots the same thing for a permanent worker fired within 10 years of becoming permanent.
Figure 4. Firing and Hiring under Both Regulations. Panel A plots the probability that an unemployed is hired before a certain time $S$. Panel B plots the probability that a permanent worker is fired within 10 years as a function of the profit level at which she becomes permanent. Panel C redoes panel B but for a temporary worker fired before becoming permanent, and as a function of initial profit level. Panel D plots the average time that a worker starting at a certain profit level would remain employed in a 15 years period.
Figure 5. Alternative Single Contract and Optimal Hiring and Firing Boundaries. Panel A reports an alternative parameterization for the Single Contract that violates Condition (15), while the parameterization for the Dual remains the benchmark one. Panel B plots the hiring and firing boundaries for these two regulations.
Figure 6. Comparative Statics: Subjective Discount Rate $\delta$. Panel A plots the firing boundaries of the Single Contract at three different seniority levels ($\tau = 6$ is a permanent worker, $\tau = 2.5$ is a worker close to become permanent, $\tau = 0.5$ is a worker hired recently) for different values of $\delta$. Panel B redoes Panel A but for the Dual Labor. Panel C compares the Dual and the Single Contract.
Figure 7. Comparative Statics: Risk-Neutral Expected Profit Variation $\mu^*$. Panels A and B plot the firing boundaries of the Single Contract and the Dual Labor, respectively, for different values of $\mu^*$ as a function of seniority at the job. Panel C plots the difference between the Dual and the Single.
Figure 8. Comparative Statics: Risk-Neutral Volatility of Profit Variation $\sigma^*$. Panels A and B plot the firing boundaries of the Single Contract and of the Dual Labor for different values of $\sigma^*$ as a function of seniority at the job. Panel C plots different firing boundaries of the Dual Labor for a parameterization with infinite costs of firing a permanent worker.
Figure 9. Comparative Statics: Risk-Aversion Parameter $\gamma$. Panel A and B plot the firing boundaries of the Single Contract and of the Dual Labor, respectively, with respect to the risk aversion parameter for two different seniority levels. Panel C plots the difference between both regulations.