Does Child Labor Decrease When Parental Incomes Rise?

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ABSTRACT

In the presence of two-sided altruism, i.e., when parents and children care about each other’s utility, increases in parental income need not always lead to increases in schooling and to decreases in child labor. This surprising result derives from the systematic way capital market constraints bind as parental income rises: child labor increases as soon as parental income rises by enough to eliminate transfers from children to parents.

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I. Introduction

In an influential paper, Baland and Robinson (2000: *BR*) address the important question of the economic efficiency of child labor. Their fundamental finding is that child labor can be inefficient in the presence of capital market constraints, i.e., when parental savings, parental bequests to children, and children’s transfers to their parents are restricted not to be negative.

In this paper, we identify a surprising property of the *BR* model: in the presence of two-sided altruism, i.e., when parents and children care about each other’s utility, some households with higher incomes will send their children to school for fewer hours, and to work for more hours, than households with lower incomes. Increases in parental income need not always lead to increases in schooling and to decreases in child labor.

That low parental income is a proximate cause of child labor is an idea that is almost universally accepted in both the theoretical literature on child labor and in more policy-oriented work.\(^1\) However, household-level empirical studies often fail to find an inverse link between household incomes and child labor.\(^2\) Our results may offer a reason why: the relationship between child labor and parental income may be neither continuous nor monotonically decreasing. Although there is no empirical work that directly examines this possibility, recent literature on transfers from adult children to their parents in developing countries has found transfer behavior that is quite consistent with the implied behavior of transfers in this model. We believe that the bridge between child labor and intergenerational transfer behavior might be an important one to explore.

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\(^2\) For surveys of this literature, see Edmonds (2002) and Dar et al (2002).
further for researchers and policymakers interested in knowing what works in addressing the problem of child labor.

The next section outlines the BR model and presents our main result. Section III shows how the main result works in a parametric example. Section IV relates our findings to the empirical literatures on child labor and on transfers from children to parents. Section V concludes.

II. The Model

We begin with a slightly modified version of the BR model with two-sided altruism. Every family has one parent and one child. There are two periods. In the first period, each child is a member of her parent’s household. In the second period, adult children maintain separate households from their parents. Every parent works only in the first period and supplies one unit of labor, which has a value, in efficiency units, of $A$.

Children may also work during the first period; any time they spend at work has a value, in efficiency units, of 1. Time not spent working is spent in school. Any labor income a child receives is controlled by her parent. When children become adults (in the second period) they control their own incomes. They then supply one unit of labor, which has a value in efficiency units that depends on the amount of schooling they received during the first period. As in BR, we assume the return to education is given by the function $h(e)$, which is assumed to have the properties $h(0) = 1$, $h'(e) > 0$, and $h''(e) < 0$. The single produced output good is the numeraire.

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3 This differs from their model in two ways: (1) in order to highlight the role of differing degrees of concern about each others’ welfare, we assume the utility function is the same for parent and child, and (2) we assume parental income is received only in the first period. Neither of these simplifying assumptions is essential to our results.

4 Later we will allow $A$ to vary and will refer to particular values of $A$ using lower-case $a$. 
Only the parent makes decisions in the first period. Each parent receives income
$A$ for her own labor and income $(1-e)$ for her child’s labor. The parent decides how much
time her child will spend at school $(e)$ and at work $(1-e)$, and how much of the total
income $(A + (1-e))$ to consume $(c_p^1)$, and how much to save $(s)$: $c_p^1 + s = A + 1 - e$.

In the second period, decisions are made both by the parent and by her (adult)
child. Each parent decides how much to consume $(c_p^2)$ and what size bequest $(b)$ to leave
to her child. The adult child decides how much to consume $(c_c)$ and how much to
transfer to her parent $(\tau)$. Thus, we have $c_p^2 + b = s + \tau$ and $c_c + \tau = h(e) + b$.

No family member can coerce another into transferring resources, nor is there any
mechanism in the capital market to allow for this. Therefore, bequests and transfers must
be non-negative. A further capital market imperfection is that parental savings, $s$, must
be non-negative.

Suppose the parent has the utility function $W_p = u(c_p^1) + u(c_p^2) + \delta W_c$, while the
adult child’s preferences are $W_c = \lambda W_p + u(c_c)$. $1 > \delta > 0$ ensures that the parent is
altruistic toward the child, and $1 > \lambda > 0$ ensures that the child is altruistic towards the
parent. $u(c)$ is a twice-continuously-differentiable function with $u'(c) > 0$ and
$u''(c) < 0$. With some straightforward substitutions, the child and parent utility functions
may be re-written as: $W_c = \left[ \lambda u(c_p^1) + u(c_p^2) \right] + u(c_c) \right] / (1 - \delta \lambda)$, and
$W_p = \left[ u(c_p^1) + u(c_p^2) + \delta u(c_c) \right] / (1 - \delta \lambda)$.

Recognizing that her consumption depends on her choices for savings, bequests
and education/labor for her child, a parent chooses $s$, $b$, and $e$, assuming that the child
will adjust $\tau$ in response to these choices. The child chooses the size of the transfer she
will give to her parent, conditional on the choices of education, savings, and bequests made by the parent.

To find equilibrium, we work backwards from the second period, first deriving the child’s rule for making an optimal transfer. The optimal transfer comes from the child’s first order condition:

$$\Lambda u'(c_p^2) - u'(c_e) \leq 0 \quad \text{(if } \tau = 0 \text{)}$$

To account for how the optimal transfers are affected by parental decisions, we note that

$$\frac{\partial \tau}{\partial s}, \frac{\partial \tau}{\partial b}, \text{ and } \frac{\partial \tau}{\partial e} \text{ equal zero when } (1) \text{ holds with strict inequality, and otherwise can be found by differentiating equation (1).}$$

Now consider the parent’s choice of savings, bequests, and how much education to provide for the child, given the child’s rule for making transfers. As in BR, we assume that the return function for education is such that in equilibrium, some education is always chosen, i.e., $$e > 0$$. The set of parents’ first-order conditions is thus:

$$-u'(c_p^1) + u'(c_p^2) + [u'(c_p^2) - \delta u'(c_e)] \frac{\partial \tau}{\partial s} \leq 0 \quad \text{(if } \tau < \text{, then } s = 0 )$$

$$-u'(c_p^1) + \delta u'(c_e) h'(e) + [u'(c_p^2) - \delta u'(c_e)] \frac{\partial \tau}{\partial e} = 0$$ (for e)

$$-u'(c_p^2) + \delta u'(c_e) + [u'(c_p^2) - \delta u'(c_e)] \frac{\partial \tau}{\partial b} \leq 0 \quad \text{(if } \tau < \text{, then } b = 0 )$$

An equilibrium in this model is any 4-tuple, $$\{\hat{s}, \hat{\tau}, \hat{b}, \hat{e}\}$$, that satisfies the parent’s and the child’s first-order conditions and yields the highest possible utility to each given the actions taken, or expected to be taken, by the other. Our objective is to study how

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5 Because of the possible feedback effect of parental decisions concerning $$s, e, \text{ and } b$$ on children’s decisions concerning $$\tau$$, and vice-versa, the parent’s optimization problem may not yield a unique solution.
equilibrium values of education and child labor -- respectively: \( \hat{e} \) and \( 1 - \hat{e} \) -- vary in response to changes in \( A \).

We begin by establishing that the three non-negativity constraints rule out certain equilibrium configurations of savings, bequests and transfers. First, we need not consider the possibility that both \( b > 0 \) and \( \tau > 0 \), since the behavior of the parent and the child is affected only by the net of benefits less transfers. We can, in fact, think of the parents as choosing bequests net -- or in anticipation-- of transfers, \( b - \tau \), and substitute the following condition for equation (2c):

\[
-u'(c_p^2) + \delta u'(c_e) \leq 0 \text{ (if "<", then } b - \tau \leq 0) \tag{3}
\]

Without loss of generality, we assume that when \( b > 0 \), then \( \tau = 0 \), and that when \( \tau > 0 \), then \( b = 0 \).

Second, we can rule out the possibility that \( s = 0 \) when \( \tau = 0 \): the non-negativity constraint on savings cannot bind if adult children do not make transfers to their parents.\(^6\) This has some intuitive appeal. One instance when adult children make transfers to their parents is when their “elderly” parents have no savings with which to support themselves. For the savings constraint to bind when \( \tau = 0 \) would require, by equation (2a), that

\[
-u'(A + 1 - e) + u'(-b) < 0. \tag{4}
\]

This inequality implies \( A + 1 - e < -b \), which in turn is impossible because since \( b \geq 0 \) when \( \tau = 0 \), the inequality implies \( A + 1 - e < 0 \).

We are left with four types of possible equilibrium configurations of \( b, \tau \) and \( s \):

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\(^6\) This is particularly obvious in our version of the BR model, as the parent earns income only in the first period, but it is also in any situation where parental earnings are no higher during the second period than during the first.
(i) \( \hat{b} = 0, \hat{\tau} > 0, \hat{s} = 0 \)

(ii) \( \hat{b} = 0, \hat{\tau} > 0, \hat{s} > 0 \)

(iii) \( \hat{b} = 0, \hat{\tau} = 0, \hat{s} > 0 \)

(iv) \( \hat{b} > 0, \hat{\tau} = 0, \hat{s} > 0 \)

We now associate equilibrium education and child labor with each of these outcome types. BR (Propositions 1 and 4) show that if the non-negativity constraint on savings and bequests or transfers does not bind (i.e., cases (ii) or (iv)), then the choice of \( e \) implied by equation (2b) satisfies the condition \( h'(e) = 1 \). We denote this efficient level of education by \( e^* \). From BR (Propositions 2, 3 and 5) we also infer that in cases (i) and (iii), the parent chooses inefficiently-low (\( e < e^* \)) hours of schooling and inefficiently-high child labor. In case (i), credit constraints prevent the family from making inter-temporal adjustments that allow it to reach efficiency. In case (iii), efficiency is compromised because the inter-generational transfers that are needed to bring it about do not occur.

In order to see how equilibrium education and child labor vary with parental income, we first define sets:\(^7\)

\[ A_1 = \{ a \geq 0 \mid \hat{b} = 0, \hat{\tau} > 0, \hat{s} = 0, \hat{e}(a_1) < e^* \}, \]

\[ A_2 = \{ a \geq 0 \mid \hat{b} = 0, \hat{\tau} \geq 0, \hat{s} > 0, \hat{e}(a_2) = e^* \}, \]

\[ A_3 = \{ a \geq 0 \mid \hat{b} = 0, \hat{\tau} = 0, \hat{s} > 0, \hat{e}(a_3) < e^* \}, \text{ and} \]

\[ A_4 = \{ a \geq 0 \mid \hat{b} \geq 0, \hat{\tau} = 0, \hat{s} > 0, \hat{e}(a_4) = e^* \}; \]

\(^7\) We need not assume there is a continuous range of incomes associated with each equilibrium type.
where, \( a \) stands for particular values of \( A \); \( a_i \) is any element of set \( A_i \); and \( \hat{e}(a) \) is the amount of schooling that a parent having income \( a \) will choose for her child.

For the moment assume that each of the four sets just defined is non-empty. We now proceed in two steps to show that education could decrease (and child labor could increase) with an increase in parental income. First, we use Proposition 1 to establish that set \( A_3 \) has at least some elements that are unique to it and which are greater than all elements in \( A_2 \) or \( A_1 \).

**Proposition 1:** Let \( a_{\text{min}}^4 = \min(A_4) \) and \( a_{\text{max}}^k = \max(A_k), \ k \in \{1,2\} \). There exists a positive but small number \( \varepsilon \) such that \( a_{\text{min}}^4 - \varepsilon > a_{\text{max}}^k \) and \( a_{\text{min}}^4 - \varepsilon \) is unique to \( A_3 \).

**Proof:** Appendix.

Proposition 1 embodies the fact that a child makes a transfer to her parent when her parent is relatively poor, while a parent leaves a bequest to her child when the parent is relatively well off. But since the child gives a stronger weight in her preferences to her own consumption, the level of income at which transfers are no longer positive will be strictly less than the level at which the parent begins to make bequests, and this means that there must be a range of parental incomes where neither transfers nor bequests are exchanged. In this range, parents and children essentially do not have the same views on which way the intra-family redistribution of resources should flow. Since any donation of resources to another family member is voluntary, no donations occur. So as the parental income rises, equilibrium will transition from one in which transfers are positive
(low parental incomes) to one in which both transfers and bequests are zero (higher parental incomes), to one in which bequests are positive (highest parental incomes).\textsuperscript{8}

Next, in Proposition 2, we establish that some low-income parents who anticipate that their children will help support them later in life educate their children more than some higher-income parents who do not anticipate such support from their children. That is, some of the unique elements of $A_3$ are associated with lower education levels than the education levels associated with at least some elements of sets $A_2$ or $A_1$.

**Proposition 2:** If $A_1$ or $A_2$ is non-empty, then there exists an $a_k \in A_1 \cup A_2$ and a unique $a_j \in A_3$ such that $a_k < a_j$, but $\hat{e}(a_k) > \hat{e}(a_j)$.

**Proof:** Appendix.

To parents with incomes in $A_1$ or $A_2$, the transfer represents in part an anticipated direct return or repayment to them from their adult children for prior investment in the children’s education. Parents in $A_3$ anticipate no such repayment and since they cannot finance their optimal consumption in later life without it, they may have to extract more from the children when they do not have to rely on their children’s voluntary cooperation. These parents make their children work more while young so that they can carry larger savings into their later years.

Note that in our statement of Proposition 2 we have assumed that there exists a (relatively low) parental income level such that we observe positive equilibrium transfers from the child to the parent. This need not be the case. If, for instance, the degree of altruism is very low (on both sides), then it is possible that for all non-negative values of $A$, there is no equilibrium with positive transfers. Parents always prefer to consume out

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\textsuperscript{8} The existence of a “middle” range of parental incomes for which transfers from children and bequests from parents are both zero has been pointed out by Laitner (1997), in a model that did not treat human capital accumulation or child labor.
of their own saving rather than face a very low relative level of consumption by accepting transfers. In this case, child education levels would be non-decreasing in parental income, and child labor would be non-increasing.

III. A Parametric Example

We now suppose that \( u(c) = \ln(c) \), \( h(e) = 1 + 2e^{1/2} \) (\( e^* = 1 \)), and \( \delta = \lambda < 1 \). The relationship between hours worked and parental incomes for this example is shown in Figure 1, for three different values of the altruism parameter: \( \lambda = \delta = 0.2, 0.4, \) and \( 0.6 \).

Figure 1 shows, first, that \( e \) is higher at any level of income, the higher is the level of altruism. Second, beginning with a parental income of zero, education \( (e) \) at first rises (child labor decreases) with increases in parental income. However, for each level of altruism, there is parental income level at which transfers from children cease, and there is a sharp drop in \( e \) at that income level.

The discontinuity corresponds to a switch from an equilibrium in which \( \tau > 0 \) to one in which \( \tau = 0 \). When parental income is relatively low and altruism is not too small, solutions exist to the first-order conditions in equations (2) that feature \( \tau > 0 \) and \( \tau = 0 \). The equilibrium is the solution that maximizes parental utility. Which set of solutions to the first-order condition will a parent prefer? When parental income is very low, a parent plans on the largess of her relatively well-educated adult child, expecting to share in her adult child’s relatively high income. She does this even though this means her second period consumption will be effectively determined by her child’s preferences. As first-period parental income becomes larger, the parent realizes that she can do better

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9 For \( \lambda = \delta = 0.1 \) (not shown), there is no parental income level for which children give transfers.
in terms of satisfying her own preferences than planning for her old age by anticipating
the largess of the adult child. At some income level, the parent prefers to achieve
relatively more second-period consumption by saving for it directly through making her
young child work more (school less) but subsequently not sharing in the adult child’s
earnings.

**IV. The Empirical Literature**

Our main result, the non-monotonicity of the relationship between hours of
education (and child labor) and parental income, is more than simply a theoretical
curiosity.

There is a large body of empirical work in which some measure of parental
income or wealth is entered as an explanatory variable for child labor or child
schooling.\(^{10}\) Nearly all of this literature assumes monotonicity in the relationship
between the right- and left-hand-side variables and finds a monotonically increasing
relationship between income and child schooling and a monotonically decreasing
relationship between income and child labor.\(^{11}\) These relationships are often not
statistically significant. Common reasons offered for this weak relationship include
measurement error in, or unaccounted for endogeneity of, the income variables, which
would tend to bias their marginal effect toward zero.\(^{12}\) To these explanations, we
suggests another plausible concern: specification error. Our findings indicate that the
relationship of child labor (and schooling) to parental resources is not monotonic.

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\(^{10}\) For surveys related to child labor, see Edmonds (2002) or Dar et al (2002). Behrman (1999) or Behrman

\(^{11}\) Exceptions to the assumption of monotonicity may be found in Bhalotra (2001) and Edmonds (2002)
who use non-parametric techniques. Interestingly, the relationship between income (expenditures) and
child labor in these papers is found to contain peaks and valleys, which is perhaps pertinent to the theory
we discuss.

\(^{12}\) Correcting for some common specification errors still produces income effects that are rather small
(Bhalotra 2001).
Measured marginal effects of income based on an assumption that it is would tend to be forced toward zero.

Of much more direct interest in connection with our main result is the empirical literature on transfers from children to parents. While this literature does not touch upon child labor issues, it nevertheless offers evidence that is consistent with the behavior of parents and children that comes out of the model we study in this paper.

In the model of this paper, children give transfers only when parental resources are relatively small; however, when the savings constraint is binding on parents, children may increase the amount of their transfers when parental income increases. Figure 2, which is derived from the parametric example of the previous section, shows that the amount of transfers could increase with parental income (when saving constraint is binding) or have an inverted-U shape (for parental incomes along the negatively-sloped portion, the savings constraint does not bind). Child education creates a quid pro quo arrangement: a higher transfer is made in return for higher schooling. In the literature on intra-household allocation, this type of behavior is referred to as the exchange motive for transfers [eg., Cox (1987), Cox and Rank (1992), Cox, Eser, and Jimenez (1998)].

These predictions are consistent with a growing body of empirical evidence. Lillard and Willis (1997), for instance, find strong evidence that transfers from grown Malaysian children to their parents are motivated by the “repayment hypothesis:” children give transfers to their parents to repay their parents for sending them to school. They find that the probability of a transfer is decreasing in parents’ income. They also find that, given that transfers occur, the transfers are increasing in the father’s income (they are increasing in mother’s income but the effect is not statistically significant).

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13 For surveys of other theoretical and empirical work on this topic, see Laitner (1997), Cox (1987).
These are all results that are entirely consistent with the behavior of transfers in the BR model when parents are savings-constrained.

Lucas and Stark (1995) find similar results for remittance behavior in a sample of rural households in Botswana. They find that remittances rise with the remitter’s educational attainment by a significantly greater amount when the remitter is the adult child of a member of the recipient household, and interpret this as evidence that the parent’s provision of schooling was an implicit loan to their children. Finally, they also find that the probability of receiving a remittance is decreasing in recipient income, and that conditional on remittance receipt, the amount of the remittance increases with recipient income.

Most of this work has specified the transfer function in a way that rules out the inverted-U shape that is possible in this model. A recent exception is a study of Peruvian households by Cox, Eser, and Jimenez (1998). This paper explores possible nonlinearity in the behavior of transfers by allowing recipient income to enter the transfer equation in a spline specification, and finds an inverted-U shape in the relationship between transfers from adult children to their parents and parental income.14

In short, recent empirical literature on transfers from adult children to their parents is entirely consistent with our findings in this paper. This literature finds evidence of an exchange motive determining the amount of transfers parents receive. We identify one form this quid pro quo can take (less child labor in exchange for larger transfers), and demonstrate the surprising implication that the relationship between

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14 Cox, Eser, and Jimenez (1998) and Cox and Rank (1992) find that parent’s transfers to children increase (or follow an inverted-U shape) with the child’s income. In the BR framework, this effect is ruled out by assuming children, while young, do not have the option of severing relations with their parents and supplying no child labor.
parental resources and child labor is non-monotonic. Our work raises this relationship as a serious area for further empirical scrutiny.

We conclude by noting that there is a very small empirical literature (we know of only two papers) that links parental altruism with child labor and child schooling. Its findings can also be explained in the terms of the BR model. In a study of nineteenth-century American industrial families, Parsons and Goldin (1989) find that they cannot reject the hypothesis that parents sent their children to work because they could not appropriate part of their children’s future incomes. In terms of the BR model, this behavior is consistent either with values of $\lambda$ that are close to zero, or with young parents’ income (which includes income from children working) being sufficiently high relative to their children’s anticipated adult earnings. In a more recent paper, Bhalotra (2001) finds evidence for Pakistan that transfers from parents are altruistically motivated, because child consumption increases with parental consumption. In the BR model, when parents provide bequests to their children, the same is true.

V. Conclusions

In stark contrast to the widely held view that reductions in poverty must reduce child labor, we have found, in the context of the BR model with two-sided altruism, that higher parental income can lead to an increase in child labor. This surprising result derives from the systematic way capital market constraints bind as parental income rises. Adult children raised in poor families make transfers to their elderly parents to repay them for income lost when the children, while young, were sent to school instead of to work. If preferences of parents and children are not identical, then at some sufficiently high level of parental income, both transfers and the hours of extra education the transfers
made possible cease. It is not until parents are wealthy enough to leave bequests, and therefore to enjoy utility solely from making their children better off, that education levels fully recover.

While the predictions of the model with regard to the relationship between income and child labor or education have yet to be tested, the model yields predictions consistent with empirical evidence on the flow of transfers from adult children to their parents. When adult children have high income relative to their parents, the model suggests that the children make transfers. Rising parental income decreases the probability of such a transfer; however, if the parent is constrained to have non-negative savings, which is a sufficient condition for ensuring a transfer, rising parental income is associated with a larger transfer. If the parent receives a transfer but is not savings constrained, the size of the transfer may decrease with increased incomes. A number of empirical studies provide evidence consistent with these patterns of predicted behavior.

In the end then, this paper makes three contributions. First, it rigorously demonstrates a surprising reason to question the usual assumption that increases in parental income will bring forth less child labor and more child schooling. Second, it shows that there is enough support for other predictions of the model to suggest that the possible non-monotonicity in the relationship between income and child labor or child schooling should be searched for with more empirical rigor. Finally, it establishes a bridge between the literatures on child labor and intergenerational transfers. With further scrutiny, this bridge may be particularly important for researchers and policymakers interested in determining strategies that will work in bringing about the eventual elimination of child labor and the attainment of efficient levels of education for children.
Appendix: Proofs of Propositions

Proposition 1: Let $a_{4}^{\min} = \min\{A_{4}\}$ and $a_{k}^{\max} = \max\{A_{k}\}$, $k = \{1, 2\}$. There exists a positive but small number $\varepsilon$ such that $a_{4}^{\min} - \varepsilon > a_{k}^{\max}$ and $a_{4}^{\min} - \varepsilon$ is unique to $A_{3}$.

Proof: We begin by establishing that $a_{4}^{\min} \notin (A_{1} \cup A_{2} \cup A_{3})$. To do this we first note that any equilibrium associated with $a_{4}$ is a solution to the parent’s optimization problem that does not imply binding non-negativity constraints on $b$ or $s$. In effect, the parents with incomes in $A_{4}$ face an unconstrained optimization problem. If $a_{4}$ also implies a solution to the first-order conditions associated with another $A_{i}$ ($i = 1, 2, 3$), the utility level associated with that latter solution to the first-order conditions must be strictly lower than the solution in $A_{4}$ because the latter solution is one in which either the savings or the bequests constraint binds. Thus any equilibrium solution associated with $a_{4}$ is unique.

Next, we verify that for any $a_{4}$, $\hat{b} = \frac{u''(c_{p}^{2})}{u''(c_{p}^{2}) + 2\delta u''(c_{e})} > 0$. This implies two things: (i) the only point in $A_{4}$ at which $\hat{b} = 0$ is $a_{4}^{\min} = \min\{A_{4}\}$; and, (ii) $a_{4}^{\min} > a_{i}$ ($i = 1, 2, 3$).

Now note that the first-order conditions for $b$ and $\tau$ cannot hold with equality at the same time. In particular, at $a_{4}^{\min}$, $\hat{\tau} = 0$ because the first-order condition for $\tau$ holds with strict inequality. This means that there exists a positive but small number $\varepsilon$ such that $a_{4}^{\min} - \varepsilon > a_{k}^{\max}$ and that $a_{4}^{\min} - \varepsilon$ is unique to $A_{3}$.

Proposition 2: If $A_{1}$ or $A_{2}$ is non-empty, then there exists an $a_{k}^{'} \in A_{1} \cup A_{2}$ and an $a_{3}^{'} \in A_{3}$ such that $a_{k}^{'} < a_{3}^{'}$, but $\hat{e}(a_{k}^{'}) > \hat{e}(a_{3}^{'})$.

Proof: There are two cases to consider: (i) $A_{2}$ non-empty; and, (ii) $A_{1}$ non-empty.

(i) In this case, we can take $a_{k}^{'} = a_{2}$, and, $a_{3}^{'} = a_{4}^{\min} - \varepsilon$. By Proposition 1 we know that $a_{k}^{'} < a_{3}^{'}$; while by the definitions of $A_{2}$ and $A_{3}$, we have $\hat{e}(a_{k}^{'}) = e^{*} > \hat{e}(a_{3}^{'})$.

(ii) We now consider the case now where $A_{2}$ is empty but $A_{1}$ is not. Let $\xi$ be some small but positive number arbitrarily set so that $a_{1}^{\max} + \xi$ is in the interior of $A_{1}$. Proposition 1 and ensures that such an $\xi$ exists. The remainder of the proof works toward showing that $\hat{e}(a_{1}^{\max}) > \hat{e}(a_{1}^{\max} + \xi)$.

The first-order condition for $\tau$ when $A = a_{1}^{\max} + \xi$ is:

$$-u'(a_{1}^{\max} + \xi + 1 - e - s) + \delta h'(e)u'(h(e)) = 0.$$
Meanwhile, the first-order condition for \( s \) says that parents want to smooth their consumption perfectly over periods one and two, so we have \( s = \frac{a_{1}^{\text{max}} + \xi + 1 - e}{2} \) and the above condition can be re-arranged and re-written as

\[
\delta \hat{h}'(e)u'(h(e)) = u'(\frac{a_{1}^{\text{max}} + \xi + 1 - e}{2}), \quad \text{or,} \quad \text{(a)}
\]

\[
h'(e) = \frac{1}{\delta u'(h(e))} u'(\frac{a_{1}^{\text{max}} + \xi + 1 - e}{2}). \quad \text{(b)}
\]

The first-order condition for \( e \), when \( A = a_{1}^{\text{max}} \) is:

\[
-u'(a_{1}^{\text{max}} + 1 - e) + \delta u'(h(e) - \tau)h'(e) + [u'(\tau) - \delta u'(h(e) - \tau)] \frac{\partial \tau}{\partial e} = 0 \quad \text{(c)}
\]

where, \( \frac{\partial \tau}{\partial e} = \frac{u''(h(e) - \tau)h'(e)}{\lambda u''(\tau) + u''(h(e) - \tau)} \). Recall from the child’s first-order condition that the optimal transfer requires \( u'(h(e) - \tau) = \lambda u'(\tau) \). Using this result and the explicit expression for \( \frac{\partial \tau}{\partial e} \) in (c), allows us to combine terms and re-write the first-order condition for \( e \), when \( A = a_{1}^{\text{max}} \) as:

\[
-u'(a_{1}^{\text{max}} + 1 - e) + \frac{1}{\lambda} \left[ \delta \lambda^2 u''(\tau) + u''(h(e) - \tau)\right] u'(h(e) - \tau)h'(e) = 0, \quad \text{or}
\]

\[
h'(e) = \lambda \frac{u'(a_{1}^{\text{max}} + 1 - e)}{u'(h(e) - \tau)} \frac{\lambda u''(\tau) + u''(h(e) - \tau)}{\sigma \lambda^2 u''(\tau) + u''(h(e) - \tau)} \quad \text{(d)}
\]

We now analyze expressions (b) and (d) to establish that \( \hat{e}(a_{1}^{\text{max}}) > \hat{e}(a_{1}^{\text{max}} + \xi) \).

To do this we first evaluate the right-hand sides of both expressions at \( e = \hat{e}(a_{1}^{\text{max}}) \) and show that at that value of \( e \), the right-hand side of (b) exceeds the right-hand side of (d). This is done in two steps. First, we verify that

\[
\frac{u'(a_{1}^{\text{max}} + 1 - \hat{e}(a_{1}^{\text{max}}))}{u'(h(\hat{e}(a_{1}^{\text{max}}) - \tau)} < \frac{u'(a_{1}^{\text{max}} + \xi + 1 - \hat{e}(a_{1}^{\text{max}}))}{u'(h(\hat{e}(a_{1}^{\text{max}})))};
\]

by noting that for sufficiently small \( \xi \), \( a_{1}^{\text{max}} + 1 - \hat{e}(a_{1}^{\text{max}}) > a_{1}^{\text{max}} + \xi + 1 - \hat{e}(a_{1}^{\text{max}}) \); that since \( \tau > 0 \), \( h(\hat{e}(a_{1}^{\text{max}}) - \tau) < h(\hat{e}(a_{1}^{\text{max}})) \); and recalling that \( u(.) \) exhibits diminishing marginal utility. Next, we establish that:
\[
\frac{\lambda u''(\tau) + u''(h(e) - \tau)}{\sigma \lambda^2 u''(\tau) + u''(h(e) - \tau)} < \frac{1}{\delta}.
\]

This can be verified directly:

\[
\lambda \delta [\lambda u''(\tau) + u''(h(e) - \tau)] > \delta \lambda^2 u''(\tau) + u''(h(e) - \tau), \quad \text{so} \quad (\lambda \delta - 1)u''(h(e) - \tau) > 0.
\]

Since \( \lambda \delta < 1 \) by assumption, this inequality must hold. Using the results of these two steps, and recalling that by definition (d) is satisfied when \( e = \hat{e}(a_i^{\max}) \), we determine that (b) is not satisfied. In particular,

\[
h'(\hat{e}(a_i^{\max})) < \frac{1}{\delta u'(h(\hat{e}(a_i^{\max})))} u'(\frac{a_i^{\max} + \xi + 1 - \hat{e}(a_i^{\max})}{2}), \quad \text{or}
\]

\[
\delta u'(h(\hat{e}(a_i^{\max}))) h'(\hat{e}(a_i^{\max})) < u'(\frac{a_i^{\max} + \xi + 1 - \hat{e}(a_i^{\max})}{2})
\]

Compare this last expression with (a). Since the left-hand side of this expression decreases in \( e \), while the right-hand side increases, the only way to achieve the equality necessary in (a) is for \( e \) to fall below \( \hat{e}(a_i^{\max}) \). Thus, we have \( a_i^{\max} + \xi > a_i^{\max} \) but \( \hat{e}(a_i^{\max} + \xi) < \hat{e}(a_i^{\max}) \).
Figure 1

Time in School and Parental Income

- $\lambda = \delta = 0.2$
- $\lambda = \delta = 0.4$
- $\lambda = \delta = 0.6$
Figure 2

Transfers From Children

Parental Income (a)

Transfer Amount

- $\lambda = \delta = 0.2$
- $\lambda = \delta = 0.4$
- $\lambda = \delta = 0.6$
- $\lambda = \delta = 0.8$
References


