The Macroeconomic Implications of Rising Wage Inequality in the United States*

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*We are grateful to Orazio Attanasio, Dirk Krueger, and Fabrizio Perri for help with the CEX data, and to Greg Kaplan for outstanding research assistance. Heathcote and Violante thank the National Science Foundation (Grant SEP-0418029). Storesletten thanks ESOP, supported by the Research Council of Norway. The opinions expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
Abstract
In recent decades, American workers have faced a rising college premium, a narrowing gender gap, and increasing wage volatility. This paper explores the quantitative and welfare implications of these changes. The framework is an incomplete-markets life-cycle model in which individuals choose education, intra-family time allocation, and savings. Given the observed history of the US wage structure, the model replicates key trends in cross-sectional inequality in hours worked, earnings and consumption. Recent cohorts enjoy welfare gains, on average, as higher relative wages for college-graduates and for women translate into higher educational attainment and a more even division of labor within the household.
1 Introduction

The structure of relative wages in the US economy has undergone a major transformation in the last three decades. Wage differentials between college graduates and high school graduates dropped in the 1970s, but have risen sharply since then (Katz and Autor, 1999). The wage gap between men and women has shrunk significantly (Goldin, 2006). Within narrow groups of workers defined by education, gender, and age, the distribution of wages has become much more dispersed (Juhn, Murphy, and Pierce, 1993). This increase in residual wage dispersion reflects increasing volatility in both persistent and transitory shocks (Gottschalk and Moffitt, 1994). Overall, the US wage structure has become more unequal.

What are the implications of this rise in wage inequality for the macroeconomy and for household welfare? Rising volatility may be expected to reduce welfare for risk-averse households with limited insurance. At the same time, households can potentially respond to a rise in the relative price of skilled labor services by investing in education, and to changes in relative wages for women by reallocating market work within the household. We address the critical welfare question using a calibrated macroeconomic model designed to capture both the new uninsured risks and the new opportunities associated with the changing wage structure.

Specifically, our model is a state-of-the-art version of the neoclassical growth model with incomplete markets and overlapping generations, the standard macroeconomic framework for studying distributional issues (e.g., Ríos-Rull, 1995; Huggett, 1996). In order to analyze the key dimensions of changes in the US wage structure, we incorporate four different types of workers, differentiated by gender and education. Individuals first make college enrollment decisions, and are then paired with individuals of the opposite sex to form households. Each period, the two spouses in working-age households draw idiosyncratic productivity shocks, and make joint consumption and time allocation decisions. Households do not have access to state-contingent claims, but can borrow and lend through a risk-free bond to smooth consumption.

The transformation in the wage structure is modeled through a combination of changes in the variances of idiosyncratic persistent and transitory productivity shocks, and changes in the relative prices of college- versus high-school-educated labor (the skill wage gap) and female versus male labor (the gender wage gap). In the model these relative prices are equilibrium market-clearing outcomes: they react both to exogenous shifts in factor loadings in the production technology and to endogenous changes in the relative supply of these factors. We label the
exogenous technological changes “skill-biased” and “gender-biased” demand shifts. The four distinct exogenous forces driving wage dynamics – skill- and gender-biased demand shifts, and changes in the volatility of persistent and transitory individual-specific productivity shocks – are parameterized to reproduce, respectively, the observed rise in the skill premium, the observed decline in the gender wage gap, and the observed changes in the covariance structure of individual wage residuals.\(^1\)

We begin our investigation by asking whether the calibrated model, with the changing wage structure as the input, can reproduce the salient trends in the empirical cross-sectional distributions of individual hours worked, household earnings, and household consumption – all endogenous outcomes of the model. Overall, our model is remarkably successful at accounting quantitatively for these trends.

The model accounts for three quarters of the observed rise in relative hours worked by women. The key driving force is the narrowing gender wage gap (as in Jones, Manuelli, and McGrattan, 2003). The model predicts little change in the dispersion of hours worked for men, as in the data. At the same time, the model underpredicts the observed decline in hours dispersion for women. Offsetting forces are at work here: more volatile idiosyncratic shocks tend to increase inequality in female hours, while the narrowing gender wage gap reduces inequality in hours for women toward the level for men.

The model successfully replicates the historical rise in the correlation between individual wages and individual hours worked. This rise is due in part to larger transitory shocks, inducing individuals to work more when wages are temporarily high, and in part to the narrowing gender gap: as wives’ earnings increase, shocks to husbands’ wages have smaller offsetting wealth effects on hours worked.

Finally, the model generates an increase in consumption dispersion which is less than half as large as the increase in household earnings dispersion, in line with the US evidence. Changes in the relative prices of different labor inputs tend to be permanent in nature, and thus affect inequality in earnings and consumption almost symmetrically. In contrast, changes in the variance of wage risk have a larger impact on earnings inequality than consumption inequality, reflecting self-insurance through labor supply and saving.

\(^1\)We experiment with two alternative models for expectations. In the benchmark model, agents are surprised only once, but thereafter enjoy perfect foresight about the future evolution of the wage structure. In the alternative model, agents are myopic, and at each date believe that the current wage structure will prevail forever, being repeatedly surprised as prices and shock variances change.
Krueger and Perri (2006) showed that the observed rise in US consumption inequality is large relative to a constrained efficient model with limited commitment, but small relative to that of a standard bond economy (as in Huggett, 1993). Our model gets consumption inequality right because its implicit insurance structure lies in between those two extremes: there are no explicit state-contingent assets, but shocks are mitigated through labor supply, intra-household risk-pooling, and social security in addition to the standard precautionary saving mechanism.

The finding that widening wage inequality is the key factor driving the trends in the distributions of hours and consumption across US households motivates us to assess the welfare implications of the transformation in the wage structure. We find that bigger persistent shocks imply sizeable welfare losses due to imperfect insurability, but gender-biased and especially skill-biased demand shifts are welfare improving: households can take great advantage of the opportunities presented by these demand shifts by increasing female participation and college enrollment, respectively. On average, entering the labor market in 2000 instead of facing the early 1960s wage structure leads to a welfare gain of 3.1% of lifetime consumption. However, the welfare gains vary dramatically across household types. In particular, high-school-educated couples were hit harshly by skill-biased demand shifts: under the same metric, they lose 1.9%. Expectations matter for the welfare analysis: in the alternative model in which agents do not foresee the future path of the college premium, gains are significantly smaller and turn negative between the mid 1970s and mid 1980s. The reason is that myopic agents in this period fail to anticipate the future rise in the skill premium, and thus (with hindsight) too few of them attend college.

Our finding of welfare gains challenges the conventional view that rising inequality led to large welfare losses (e.g., Attanasio and Davis, 1996; Krueger and Perri, 2004). Our welfare estimates are less pessimistic for two reasons. First, our model incorporates additional channels of behavioral adjustment in response to exogenous labor market changes. Second, our welfare estimates are derived using a structural equilibrium model that links changes in relative wages to their average level. For example, all else equal, an increase in transitory, hence insurable, wage uncertainty will increase average labor productivity and the average wage.

The rest of the paper is organized as follows. Section 2 describes the stylized facts of interest. Section 3 presents the model and defines the equilibrium. Section 4 describes the calibration and estimation strategy. Section 5 contains the main results on the macroeconomic consequences of the changing wage structure. Section 6 contains the welfare analysis and the results for the
economy with myopic beliefs. Section 7 concludes. The Appendix has additional information on micro data and sample selection, identification, and estimation of the statistical wage process, the numerical algorithm for computing the equilibrium, and an extensive comparison between the perfect-foresight and the myopic-belief economies.

2 Stylized facts

This section describes the salient facts motivating our exercise. Statistics on wages, hours, and earnings reported in this section are all computed from the Current Population Survey (CPS) March files (1967-2005). Statistics on household consumption are based on Consumer Expenditure Survey (CEX) data (1980-2003). Our sample comprises married households where the husband is 25-59 years old. We begin by describing the changes in the wage structure that serve as inputs for our model: our parameterization strategy is designed to match these facts. We then review changes in the cross-sectional distributions for hours, earnings, and consumption that serve as targets in our analysis.

2.1 Model inputs

Panel (A) of Figure 1 plots the variance of log hourly wages for men since 1967. This rise in cross-sectional wage inequality has been well documented in the literature (e.g., Heathcote, Perri, and Violante, 2010) and is the starting point of our study. Two main forces contributed to the expansion of the wage distribution: the rise in the skill (education) premium and the rise in dispersion within skill groups (Juhn, Murphy, and Pierce, 1993). In turn, the latter is due to increasing volatility in persistent and transitory shocks. We return to this in Section 4. Decomposing the overall increase in male wage dispersion, the widening college premium accounts for around 1/3 of the increase, while widening residual dispersion explains the rest.

Panel (B) plots the evolution of the college wage premium, defined as the ratio between the average hourly wage of workers with at least a college degree and the average hourly wage of workers without a college degree. The college premium declined slightly in the first

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2Section A1 of the Appendix contains a detailed description of the underlying micro data, the handling of measurement issues, and the sample selection criteria. There, we also document that cross-sectional moments computed from the Panel Study of Income Dynamics (PSID) display similar trends to their CPS counterparts, with a few exceptions that we discuss.

3The cross-sectional moments plotted in Figures 1 through 6 are demeaned in order to visualize differences in trends. Means are reported in square brackets in the legends.
part of the sample period, but has been rising since the late 1970s. Panel (C) plots college completion rates over the same period, defined as the fraction of 25- to 29-year-olds with a college degree. Completion rates rose dramatically over the sample period, especially for women: only 12% of women in this age group had a college degree in 1967, compared to 32% in 2005. The simultaneous increase in college completion rates and the college wage premium indicates growth in aggregate labor demand favoring college graduates, which, following the literature, we label a “skill-biased demand shift.” In the existing literature the leading explanation for this shift is the rapid adoption of new information and communication technologies (ICT), which raised the relative productivity of more educated labor (“skill-biased technical change”).

Panel (D) of Figure 1 depicts the dynamics of the ratio of female to male wages. This ratio was constant until the late 1970s and increased thereafter, implying a significant narrowing of the gender wage gap. As is well known, female labor force participation (a model target; see panel (A) of Figure 2) increased sharply over this same period. We interpret this concurrent growth in relative price and relative supply of female labor symmetrically with college-educated labor, and conclude that a “gender-biased demand shift” in favor of female labor was operative over this period. This shift could be driven by changes in technology favoring services occupations in which women have a comparative advantage (Johnson and Keane, 2007), or by changes in social norms making qualified women more willing to seek high-paying positions, and employers more willing to hire them (Goldin, 2006).

### 2.2 Model targets

The changes in the wage structure described above are inputs for our quantitative exercise. Our goal is to assess whether these changes can account for the key targets of our theory, namely, the observed changes in the distributions of male and female hours worked (gender differentials in average hours, variances of hours, and wage-hour correlations), in household

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4 A less prominent role is attributed to falling demand for unskilled-intensive goods produced in the US due to greater openness to trade, and to changes in labor market institutions such as declining union power. See, for example, Katz and Autor (1999), Acemoglu (2002), and Hornstein, Krusell, and Violante (2005) for surveys.

5 As is common in the literature, we report the full-time gender gap, where full-time work is defined to be 2,000 hours per year or more. This criterion is used because women are more likely to be employed part-time, and part-time work carries a wage penalty (see, e.g., Blau and Kahn, 2000).

6 Goldin (2006) discusses the sources of this demand shift – what she calls the “quiet revolution.” She points to the impact of World War II in showing employers that women could be profitable and reliable workers; the role of contraceptives in allowing women to plan their careers and to become viable candidates for high-paying jobs; the structural shift toward the service sector with its more flexible work schedule; and, finally, the role of antidiscrimination legislation.
earnings inequality, and, finally, in household consumption inequality.

Panel (A) of Figure 2 plots the ratio of female to male market hours, and shows the well-known rise in female labor market participation: in the late 1960s women worked 30% as much as men, while since 2000 women’s market hours have been around two thirds of men’s.

Panel (B) of Figure 2 plots the variance of log hours worked within groups defined by gender. There is much more dispersion in hours worked for women than for men, and the variance of female hours declines throughout the period, while the corresponding series is basically flat for men. Panel (C) reports the cross-sectional correlation between log wages and log hours by gender. This correlation rises until the late 1980s. The rise for men is more pronounced, around 0.25 versus 0.15 for women. In the 1990s and beyond, the correlation is stable for men, while it declines somewhat for women.

The variances of household log earnings and equivalized log consumption are plotted in panel (D). Household earnings inequality rose steadily by 23 log points over the period, driven by increases in wage inequality and in the wage-hour correlation. The second line in panel (D) is the variance of log household equivalized consumption. The CEX data, assembled by Krueger and Perri (2006), are consistently available only since 1980. Consumption (labeled CEX ND+) is defined as expenditures on nondurable goods, services, and small durables, plus services from housing and vehicles. Consumption inequality tracks earnings inequality closely in the 1980s (Cutler and Katz, 1991), while the two series diverge in the 1990s and beyond (Slesnick, 2001; Krueger and Perri, 2006).

Overall, between 1980 and 2003, household log earnings dispersion rises more than twice as much as log consumption dispersion: 17 versus 7 log points. Comparable results on trends in US consumption inequality for the 1990s are reported by Attanasio, Battistin, and Ichimura (2007), and Blundell, Pistaferri, and Preston (2008), notwithstanding differences in the methodologies used to organize the data.

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7 By construction, this statistic excludes nonparticipants. We define an individual as a nonparticipant if he/she works less than 13 weeks at 20 hours per week, i.e., a quarter of part-time employment. None of the key trends in hours is sensitive to this threshold; however, the lower the threshold, the higher is the level of measured inequality.

8 Following Krueger and Perri (2006), we also use the Census scale to construct adult equivalent measures of household consumption. We do not equivalize earnings, but this choice is largely innocuous: the increase in the variance of household log equivalized earnings is just 3 log points lower than that of the unequivalized series.
3 Economic model

We begin by describing the model's demographic structure, preferences, production technologies, government policies, and financial markets. Next, we outline the life cycle of the agents and define a competitive equilibrium.

3.1 Preliminaries

Time is discrete, indexed by \( t = 0, 1, \ldots \), and continues forever. The economy is populated by a continuum of individuals, equally many males and females. Gender is indexed by \( g \in \{m, f\} \) and age by \( j \in J \equiv \{1, 2, \ldots, J\} \). Individuals survive from age \( j \) to \( j + 1 \) with probability \( \zeta^j \). At each date a new cohort of measure one of each gender enters the economy. Since cohort size and survival probabilities are time-invariant, the model age distribution is stationary.

The life cycle of individuals comprises four stages: education, matching, work, and retirement. In the first two stages, the decision unit is the individual. In the second two, the decision unit is the household, i.e., a husband and wife pair. Since our focus is mostly on labor market risk, we simplify the first two stages by letting education and matching take place sequentially in a pre-labor-market period of life labeled age zero. Agents enter the labor market as married adults at age \( j = 1 \), retire at age \( j = j^R \), and die with certainty if they reach age \( j = J \).

We adopt a unitary model where both members of a household have common preferences \( u(c, n^m, n^f) \), where \( c \geq 0 \) is market consumption, and \( n^g \in [0, 1] \) is market hours of the spouse of gender \( g \). The assumption that the husband’s and wife’s utilities coincide can be interpreted in several ways. One interpretation is that male and female nonmarket time produces a public home-consumption good and that market consumption is also public. Alternatively, it could be that consumption and nonmarket time are private goods but household members are perfectly altruistic toward each other.

The consumption good is produced by a representative firm using aggregate capital \( K_t \) and an aggregate labor input \( H_t \) according to a Cobb-Douglas production technology \( Z_t K_t^\alpha H_t^{1-\alpha} \), where \( \alpha \) is capital’s share of output, and \( Z_t \) is a time-varying scaling factor. Output can be used for household consumption \( C_t \), government consumption \( G_t \), investment \( I_t \), or net exports \( NX_t \). Capital depreciates at rate \( \delta \).

We follow Katz and Murphy (1992) and Heckman, Lochner, and Taber (1998) in modelling aggregate labor \( H_t \) as a constant elasticity of substitution aggregator of four types of labor
input, $H_{i}^{g,e}$, indexed by gender $g$ and education level $e \in \mathcal{E} \equiv \{h, l\}$, where $h$ denotes high education and $l$ low education:

$$H_i = \left[ \lambda_i^S \left( \lambda_i^G H_i^{f,h} + (1 - \lambda_i^G) H_i^{m,h} \right)^{\frac{\sigma+1}{\sigma}} + (1 - \lambda_i^S) \left( \lambda_i^G H_i^{l,l} + (1 - \lambda_i^G) H_i^{m,l} \right)^{\frac{\sigma+1}{\sigma}} \right]^{\frac{1}{\sigma}}. \quad (1)$$

According to this specification, male and female efficiency units of labor, conditional on sharing the same education level, are perfect substitutes, while the elasticity of substitution between the two different education groups is $\theta$. In Section 2 we interpreted the simultaneous increases in the prices and quantities of college-educated and female labor as reflecting skill-biased and gender-biased demand shifts. In the aggregator above, these demand shifts are captured, respectively, by the variables $\lambda_i^S$ and $\lambda_i^G$.

Financial markets are incomplete: agents trade risk-free bonds, subject to a borrowing constraint, but cannot buy state-contingent insurance against individual labor-income risk. The interest rate on the bonds is set internationally and is assumed to be constant and equal to $r$. Agents can also buy annuities at actuarially fair rates. All markets are competitive.

The government levies flat taxes ($\tau^n, \tau^a$) on labor and asset income and runs a public pension system which pays a fixed benefit $b$ to retirees. Once the pension system has been financed, any excess tax revenues are spent on nonvalued government consumption $G_t$.

### 3.2 Life cycle

We now describe the four stages of the life cycle in detail.

#### 3.2.1 Education

At the start of life (age zero), individuals make a discrete education choice between pursuing a college degree ($e = h$) or a lower schooling degree ($e = l$). The utility cost of attending

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9 Estimates of the elasticity of substitution between equally skilled individuals of different gender are high. For example, Johnson and Keane (2007) estimate an elasticity above five for men and women in the same education/occupation/age group.

10 The term $(1 - \lambda_i^G)/\lambda_i^G$ creates a time-varying wedge between the wages of men and women with the same human capital. Jones, Manuelli, and McGrattan (2003) model this wedge as a “tax” on the female wage in the household budget constraint. They calibrate this sequence by matching the observed gender premium, exactly as we do. From the viewpoint of an agent in the model, these alternative modelling strategies are equivalent.

11 In an earlier version of the paper, we explored a closed economy version of the model, with an endogenous time-varying interest rate. The differences between the closed and open economy versions of the model turned out to be quantitatively negligible.

12 This allows us to abstract from bequests. Since bequests are typically received at ages when wealth is already sizeable, they are not an important insurance channel against income shocks.
college $\kappa$ is idiosyncratic, and is drawn from the gender- and cohort-specific distribution $F^g_t (\kappa)$. This distribution captures, in reduced form, cross-sectional variation in the psychological and pecuniary factors that make acquiring a college degree costly, such as variation in pure scholastic talent, tuition fees, parental resources, access to credit, and government aid programs.

When individuals decide whether or not to go to college, they consider their draw for the cost, $\kappa$, the college wage premium they expect to get in the labor market, and the value of being highly educated when entering the matching stage: with positive assortative matching, acquiring a college education increases the probability of meeting a college-educated, and thus high-earning, spouse. Let $M^g_t (e)$ be the expected value, upon entering the matching stage at date $t$, for an individual of gender $g$ who has chosen education level $e$. The optimal education choice for an individual of cohort $t$ with education cost $\kappa$ is

$$
e^g_t (\kappa) = \begin{cases} h & \text{if } M^g_t (h) - \kappa \geq M^g_t (l), \\ l & \text{otherwise} \end{cases}$$

where $e^g_t (\cdot)$ denotes the gender-specific education decision rule.\footnote{Our simple model for education acquisition is consistent with several key empirical patterns: (i) a positive correlation between education and scholastic ability/parental background (i.e., low $\kappa$), (ii) a positive correlation between education and wages, and, therefore, (iii) a positive correlation between measures of ability/background and wages. In the model, $\kappa$ does not have a direct effect on earnings; it impacts earnings only through education. The debate on whether there are returns to ability above and beyond education is ongoing. For example, Cawley, Heckman, and Vytlacil (2001) argue that measures of cognitive ability and schooling are so strongly correlated that one cannot separate their effects on labor market outcomes without imposing arbitrary parametric structures in estimation (e.g., log-linearity and separability) which, when tested, are usually rejected.}

Let $q^g_t$ be the fraction of individuals of gender $g$ choosing to attend college in period $t$. Then

$$q^g_t = F^g_t (M^g_t (h) - M^g_t (l)) \in [0, 1].$$

**3.2.2 Matching**

Upon entering the matching stage, individuals are characterized by two states: gender and education $(g, e)$. Following Fernández and Rogerson (2001), individuals of opposite gender are matched stochastically based on their educational level. Let $\pi^m_t (e^m, e^f) \in [0, 1]$ be the probability that a man in education group $e^m$ is assigned to a woman belonging to group $e^f$ at time $t$. Symmetrically, matching probabilities for women are denoted $\pi^f_t (e^f, e^m)$.

The expected values upon entering the matching stage for men of high and low education levels can be written, respectively, as

$$
M^m_t (h) = \pi^m_t (h, h) \Psi^0_t (h, h) + \pi^m_t (h, l) \Psi^0_t (h, l),
$$

$$
M^m_t (l) = \pi^m_t (l, l) \Psi^0_t (l, l) + \pi^m_t (l, h) \Psi^0_t (l, h),
$$
where $V_0^t (e^m, e^f)$ is expected lifetime utility at date $t$ for each member of a newly married (age zero) couple comprising a male with education $e^m$ and a female with education $e^f$. Similar expressions can be derived for the functions $M_0^f (e)$.

The enrollment rates from the schooling stage, $q_0^g$, together with the matching probabilities, $\pi_0^g$, jointly determine the education composition of newly formed households. For example, the fraction of matches of mixed type $(h, l)$ at date $t$ is given by

$$q_t^m \pi_t^m (h, l) = \left( 1 - q_t^f \right) \pi_t^f (l, h),$$

where the equality is an aggregate consistency condition. Since all individuals end up in married couples, the constraint

$$\pi_t^g (e^g, h) + \pi_t^g (e^g, l) = 1$$

must hold for all pairs $(g, e^g)$.

One can show that the cross-sectional Pearson correlation between the education levels of husband and wife, a measure of the degree of assortative matching in the economy, is given by

$$\rho_t = \frac{q_t^m \pi_t^m (h, h) - q_t^m q_t^f}{\sqrt{q_t^m (1 - q_t^m) q_t^f \left( 1 - q_t^f \right)}}.$$

We treat this correlation as a structural parameter of the economy, and for simplicity we restrict it to be time-invariant, i.e., $\rho_t = \rho^*$ for all $t$. Finally, since our focus is on labor market risk, we abstract from shocks to family composition: matching takes place only once, and marital unions last until the couple dies together.$^{14}$

### 3.2.3 Work and retirement

Individuals start working at age $j = 1$ and retire at age $j^R$. An individual’s endowment of efficiency units per hour of market work (or individual labor productivity) depends on experience and on the history of idiosyncratic labor productivity shocks. Thus, at time $t$, the hourly wage for an individual of age $j$ and type $(g, e)$ is

$$p_{t}^{g,e} \times \exp (L (j) + y_t),$$

$^{14}$See Cubeddu and Ríos-Rull (2003) for a quantitative investigation of the role of shocks to family composition on aggregate saving, wealth inequality, and other macroeconomic magnitudes.
where $L(j)$ is a deterministic function of age and $y_t$ is the stochastic individual-specific component of (log) labor productivity.\footnote{Our model assumes a return to age rather than to actual labor market experience. This choice is made out of convenience: accounting explicitly for the return to experience would add two continuous state variables (one for each spouse), making the problem significantly harder to solve. This simplification is unlikely to matter for men’s choices, since the vast majority participate throughout the working life anyway. In the literature there are different views on the role of labor market experience for women’s work decisions. Olivetti (2006) argues that increases in returns to experience have had a large effect on women’s hours worked in the last three decades. In contrast, Attanasio, Low and Sánchez-Marcos (2008) find small effects.}

Men and women face the same experience profile and the same stochastic process for idiosyncratic productivity. We model $y_t$ as the sum of two orthogonal components: a persistent autoregressive shock and a transitory shock. More precisely,

$$
y_t = \eta_t + v_t, \quad \eta_t = \rho \eta_{t-1} + \omega_t,
$$

where $v_t$ and $\omega_t$ are drawn from distributions with mean zero and variances $\lambda^v_t$ and $\lambda^\omega_t$, respectively. The sequences $\{\lambda^v_t, \lambda^\omega_t\}$ capture time variation in the dispersion of idiosyncratic transitory and persistent shocks. The initial ($j = 1$) value for the persistent component is drawn from a time-invariant distribution with mean zero and variance $\lambda^v$. Shocks are positively but imperfectly correlated across spouses within a household. In what follows, for notational simplicity, we stack the two idiosyncratic components $\{\eta_t, v_t\}$ for an individual of gender $g$ in the vector $y^g_t \in \mathcal{Y}$, and denote her/his individual efficiency units by $\varepsilon(j, y^g_t)$. We discuss all these modelling choices for the wage process in Section A2 of the Appendix.

Household holdings of the risk-free asset are denoted $a_t \in \mathcal{A} \equiv [a, \infty)$, where $a$ is the borrowing limit. One unit of savings delivers $1/\zeta^j$ units of assets next period, reflecting the annuity-market survivors’ premium.

The problem of a working household can thus be written as follows:

$$
\forall_t \left( e^m, e^f, j, a_t, y^m_t, y^f_t \right) = \max_{c_t, a_{t+1}, n^m_t, n^f_t} u \left( c_t, n^m_t, n^f_t \right) +
+ \beta \zeta^j E_t \left[ \forall_{t+1} \left( e^m, e^f, j+1, a_{t+1}, y^m_{t+1}, y^f_{t+1} \right) \right]
$$

subject to

$$
c_t + \zeta^j a_{t+1} = [1 + (1 - \tau^a) r] a_t + (1 - \tau^a) \left[ p^m_{t} \varepsilon (j, y^m_t) n^m_t + p^f_{t} \varepsilon (j, y^f_t) n^f_t \right]
$$

$$
a_{t+1} \geq a, \quad c_t \geq 0, \quad n^m_t, n^f_t \in [0, 1],
$$

where $\varepsilon(j, y^m_t)$ and $\varepsilon(j, y^f_t)$ are the individual-specific efficiency units.
where the value function $V_t$ defines expected discounted utility at time $t$ as a function of the state variables for the household problem: education $(e^m, e^f)$, age $j$, wealth $a_t$, and the vectors of male and female productivity $(y_t^m, y_t^f)$. Preferences and the asset market structure imply that there are neither voluntary nor accidental bequests.

The expected lifetime value for each spouse in a newly formed household, $V_0^0$, is given by

$$V_0^0(e^m, e^f) = E \left[ V_t \left( e^m, e^f, 1, 0, y_t^m, y_t^f \right) \right],$$

where the zero value for the fourth argument reflects the assumption that agents enter the working stage of the life cycle with zero wealth, and where the expectation is taken over the set of possible productivity realizations at age one.\(^\text{16}\)

The maximization problem for retirees is identical to the workers’ problem (10), with two exceptions: (i) labor supply $n_t^q$ is constrained to be zero, and (ii) each period retired individuals receive a lump-sum public transfer $b$, taxed at rate $\tau^m$.

### 3.3 Equilibrium

The economy is initially in a steady-state. Unexpectedly, agents discover that the economy will experience a period of structural change, driven by the sequences for skill-biased and gender-biased demand shifts and the variances of the stochastic wage components $\{\lambda_t\} \equiv \{\lambda_t^S, \lambda_t^G, \lambda_t^v, \lambda_t^w\}$, and by the sequences for TFP and education cost distributions $\{Z_t, F_t^0\}$. After the initial announcement, agents have perfect foresight over these sequences.\(^\text{17}\)

Let $\mathcal{B}_A$ and $\mathcal{B}_Y$ be the Borel sigma algebras of $A$ and $Y$, and $P(\mathcal{E})$ and $P(\mathcal{J})$ be the power sets of $\mathcal{E}$ and $\mathcal{J}$. The state space is denoted by $S \equiv \mathcal{E}^2 \times \mathcal{J} \times A \times Y^2$. Let $\Sigma_S$ be the sigma algebra on $S$ and $(S, \Sigma_S)$ the corresponding measurable space. Denote the measure of households on $(S, \Sigma_S)$ in period $t$ as $\mu_t$ and the initial stationary distribution as $\mu_*$.\(^\text{18}\)

Given $\mu_*$ and sequences $\{\lambda_t\}$ and $\{Z_t, F_t^0\}$, a competitive equilibrium is a sequence of discounted values $\{M_t^0(e)\}$; decision rules for education, consumption, hours worked, and savings $\{e_t^0(\kappa), c_t(s), n_t^q(s), a_{t+1}(s)\}$; value functions $\{V_t(s)\}$; firm choices $\{h_t^{q,e}, K_t\}$; prices $\{\pi_t^{q,e}\}$; government expenditures $\{G_t\}$; individual college enrollment rates by gender and cohort $\{q_t^g\}$; matching probabilities $\{\pi_t^q\}$; and measures of households $\{\mu_t\}$ such that, for all $t$:

\(^{16}\)The assumption of zero initial wealth is consistent with the absence of bequests in equilibrium. We analyzed the empirical distribution of financial wealth for individuals aged 23-25 in the United States from the 1992 Survey of Consumer Finances. We found that median wealth is negligible for this age group ($\$2,000$), with no significant differences across the two education groups. Details are available upon request.

\(^{17}\)In Section 6.5 we study an alternative economy where agents hold myopic beliefs about these sequences.
1. The education decision rule \( q^g_t (\kappa) \) solves the individual problem (2) and \( q^g_t \) is the fraction of college graduates of gender \( g \) determined by (3).

2. The matching probabilities \( \pi_i^g \) satisfy the consistency conditions in (5) and (6), and are consistent with the target degree of assortative matching \( p^* \) in (7). Moreover, the discounted utilities at this stage, \( M^g_t (e) \), are defined in (4).

3. The decision rules \( c_t (s), n_i^g_t (s), a_{t+1} (s) \) and value functions \( \mathbb{V}_t (s) \) solve the household problem (10) during the work stage, and the analogous problem during retirement.

4. Capital and labor inputs are allocated optimally, i.e., \( K_t \) and \( H_{t}^{g,e} \) satisfy

\[
 r = \alpha Z_t \left( \frac{H_t^c}{K_t} \right)^{1-\alpha} - \delta, \quad p_t^{m,h} = \Omega_t^h \left( 1 - \lambda_t^G \right) \lambda_t^S, \quad p_t^{m,l} = \Omega_t^l \left( 1 - \lambda_t^G \right) \left( 1 - \lambda_t^S \right), \quad p_t^{f,h} = \Omega_t^h \lambda_t^G \lambda_t^S, \quad p_t^{f,l} = \Omega_t^l \lambda_t^G \left( 1 - \lambda_t^S \right),
\]

where \( \Omega_t^h \equiv (1 - \alpha) Z_t \left( \frac{K_t}{H_t^c} \right)^\alpha H_t^{1-\alpha} \left( \lambda_t^G H_t^{f,c} + (1 - \lambda_t^G) H_t^{m,c} \right)^{-1/\alpha} \) and \( H_t \) is given by (1).

5. The domestic labor markets clear, i.e., for all \((g, e)\) pairs, \( H_t^{g,e} \) satisfies

\[
 H_t^{g,e} = \int_{S \in \mathbb{E}} P_{t+j}^{g,e} d\mu_t = \tau^a r A_t + \tau^m \sum_{g,e} p_t^{g,e} H_t^{g,e}.
\]

6. The domestic good market clears, \( C_t + K_{t+1} - (1 - \delta) K_t + G_t + N X_t = Z_t K_t^\alpha H_t^{1-\alpha} \), where \( C_t = \int_S c_t (s) d\mu_t \) is aggregate consumption.

7. The world asset market clears. This requires that the change in net foreign asset position between \( t \) and \( t+1 \) equals the year-\( t \) current account: \((A_{t+1} - K_{t+1}) - (A_t - K_t) = N X_t + r (A_t - K_t)\), where \( A_{t+1} = \int_S a_{t+1} (s) d\mu_t \) is aggregate domestic wealth.

8. The government budget is balanced, \( G_t + (1 - \tau^n) b \int_{S \in \mathbb{J}} d\mu_t = \tau^a r A_t + \tau^n \sum_{g,e} p_t^{g,e} H_t^{g,e} \).

9. The sequence of measures \( \{\mu_t\} \) is consistent with household decision rules. For all \( s \equiv (e^m, e^f, j, a_t, y^m_t, y^f_t) \in \mathbb{S} \), and \( \mathbb{S} \equiv (\mathbb{E}^m \times \mathbb{E}^f \times \mathbb{J} \times \mathbb{A} \times \mathbb{Y}^m \times \mathbb{Y}^f) \in \Sigma_S \), where \( \{1\} \notin \mathbb{J} \), the measures \( \mu_t \) satisfy \( \mu_{t+1} (\mathbb{S}) = \int_{\mathbb{S}} Q_t (s, \mathbb{S}) d\mu_t \) with

\[
 Q_t (s, \mathbb{S}) = \mathcal{I}_{\{e^m \in \mathbb{E}^m, e^f \in \mathbb{E}^f, j+1 \in \mathbb{J}, a_{t+1} (s) \in \mathbb{A}\}} \Pr \left\{ y^m_{t+1} \in \mathbb{Y}^m, y^f_{t+1} \in \mathbb{Y}^f | y^m_t, y^f_t \right\} \zeta^j.
\]

The initial measure at age \( j = 1 \) for the \((h, h)\) type is obtained as

\[
 \mu_t \left( \{h\}, \{h\}, \{1\}, \{0\}, \mathbb{Y}^m, \mathbb{Y}^f \right) = q^m_t \pi^m_t (h, h) \Pr \left\{ y^m_t \in \mathbb{Y}^m, y^f_t \in \mathbb{Y}^f | j = 1 \right\},
\]

and so on for all other education pairs.
4 Parametrization

We now turn to the calibration of the model. We begin with the parameters set outside the model (Sections 4.1-4.2), and then move to those whose calibration requires solving for equilibrium allocations (Sections 4.4-4.3). Table 1 summarizes the calibration strategy and parameter values. Section A3 of the Appendix outlines the computational algorithm for solving the model economy.

4.1 Demography and technology

The model period is one year. After schooling choice and household formation, individuals enter the labor market at age 25 (model age \( j = 1 \)), which is the median age of first marriage for men in 1982, the midpoint of our sample. They retire on their 60th birthday, which implies \( j^R = 35 \), and die by age 100, so \( J = 75 \). Mortality probabilities \( \{\zeta^j\} \) are from the 1992 US Life Tables of the National Center for Health Statistics.

Turning to the aggregate technology, we follow Katz and Murphy (1992) in setting the parameter \( \omega \) measuring the elasticity of substitution between education groups to 1.43. The constant world pre-tax interest rate \( r \) is set to 5%. Capital's share \( \alpha \) is set to 0.33 and the depreciation rate \( \delta \) to 0.06, so the capital-to-output ratio \( K/Y = \alpha/(r + \delta) = 3 \). Following Domeij and Heathcote (2004), the tax rates on labor and capital income are set to \( \tau^n = 0.27 \) and \( \tau^a = 0.40 \), which implies an after-tax return to saving of 3%.

4.2 Idiosyncratic productivity shocks

The mapping between observed individual hourly wages and individual labor productivity is not immediate in our model, for two reasons. First, as is clear from equation (8), one must filter out from observed wages changes in equilibrium prices \( p^0,e \) to isolate changes in efficiency units. Second, an individual’s wage is observed in the data only if she/he works enough hours (a quarter of part-time employment) to qualify for inclusion in our sample. This selection problem is acute for women, especially in the first part of the sample period. Since in the model men and women are assumed to face the same stochastic process for labor productivity shocks, the process can be estimated using only wage data for men, for whom selection is not a major concern.\(^{18}\)

\(^{18}\)Low, Meghir, and Pistaferri (forthcoming) provide evidence on this. Attanasio et al. (2008) make the same symmetry assumption and find that it implies the right magnitude for the female wage variance, under the
Let $w_{i,j,t}$ be the hourly wage of individual $i$ of age $j$ at time $t$. Using PSID data, we run an ordinary least-squares regression of male hourly wages on a time dummy, a time dummy interacted with a college education dummy ($e_i$), and a cubic polynomial in potential experience (age minus years of education minus five) $L(j)$:

$$\ln w_{i,j,t} = \beta_0^t + \beta_1^t e_i + L(j) + y_{i,j,t}.$$  \hspace{1cm} (12)

This specification is consistent with the wage equation (8) in the structural model. The residuals of equation (12) are a consistent estimate of the stochastic labor productivity component, since education is predetermined with respect to the realizations of $y_{i,j,t}$.

As described in equation (9), $y_{i,j,t}$ is modeled as the sum of a transitory plus a persistent component, with time-varying variances—a necessary feature to capture trends in residual wage dispersion. Since one cannot separately identify the variance of the genuine transitory shock from the variance of measurement error, we assume that the variance of measurement error is time-invariant, and use an external estimate. Based on the PSID Validation Study for 1982 and 1986, French (2004) finds a variance of measurement error in log hourly wages of 0.02. Expressed as a percentage of the residual wage variance in our sample, measurement error accounts for 8.5% of the total. Our estimation method is designed to minimize the distance between model and data with respect to the variances and covariances of wage residuals across cells defined by year and age. In Section A2 of the Appendix, we motivate the specification in (9) for the wage process, and discuss its identification and estimation in detail.

Our findings are summarized in Figure 3. Panel (C) shows that the conditional variance of persistent shocks $\lambda^{\omega}_t$ doubles during the 1975-1985 decade. The point estimate for the initial (age 1) variance of the persistent component $\lambda^{\omega}$ is 0.124, and shocks to this component are very persistent: the estimated annual autocorrelation coefficient $\rho$ is 0.973. Panel (D) of Figure 3 displays the variance of the genuine transitory shocks $\lambda^v_t$ (i.e., the uncorrelated component of residual log wages, net of measurement error). This variance grows over time. Panels (C)-(D) also plot bootstrapped standard errors of the estimates. In general, standard errors are small and the trends significant. As inputs for the model, we use Hodrick-Prescott (HP) filtered trends of the estimated sequences $\{\lambda^v_t, \lambda^{\omega}_t\}$, with an HP smoothing parameter of ten.

The remaining aspect of the wage process is the correlation structure for shocks within the household. The correlation between husband and wife in the initial persistent productivity draw model's selection mechanism. As documented in Heathcote et al. (2008a), our model has the same implication.
\( \eta_0 \) is set equal to the empirical correlation of education levels in our PSID sample for newly formed households (aged 25-35), which is 0.517.\(^{19}\) The cross-spouse correlations for transitory shocks and persistent shocks are set to a common value that reproduces, in equilibrium, the average observed correlation between wage growth for husbands and wives. This empirical correlation, corrected for measurement error, is 0.15, which the model replicates when setting, as a structural parameter, the shock correlation to 0.134.\(^{20}\)

### 4.3 Demand shifts, TFP, and information

The sequences \( \{\lambda^S_t, \lambda^G_t\} \) are set to ensure that the equilibrium model time paths for the male college wage premium and for the gender wage gap match their empirical counterparts, where these trends are defined by applying an HP filter with a smoothing parameter of ten to the raw PSID data. Panels (A) and (B) of Figure 3 show that the implied paths for \( \lambda^S_t \) and \( \lambda^G_t \) are qualitatively similar to those for the skill premium and the gender gap presented in Figure 1.

We set the path for the aggregate scaling factor \( Z_t \) so that, in the absence of any behavioral response (i.e., assuming no changes in total effective hours for each type of labor input), the dynamics of \( \lambda_t \) would leave average output and labor productivity constant at the initial steady-state levels. We make this choice because we want to remain agnostic about the precise microfoundations underlying the dynamics in the components of \( \lambda_t \), and thus we want to avoid hardwiring aggregate productivity changes in a particular direction into the design of the experiment. It could be that some of the forces that have caused the observed dynamics in \( \lambda_t \) – e.g., the fall in the price of ICT capital for skill-biased demand shifts – have also directly increased economy-wide TFP and thus welfare. Any such gains would need to be added to the behavior-induced effects that we quantify below.

Finally, in our benchmark economy agents learn about the changing wage structure in 1965, and from then on they have perfect foresight over \( \{\lambda_t\} \) and over \( \{Z_t, F^S_t\} \).

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\(^{19}\)Our preferred interpretation for this assumption is that when matching, agents sort positively with respect to wages, irrespective of whether wage differences reflect education or the initial draw for the persistent component. The initial persistent draw does not appear explicitly in our expressions for matching probabilities, but sorting in this dimension is implicit in expected match values.

\(^{20}\)These two choices for within-household shock correlation are supported by existing studies. Hyslop (2001, Table 3) estimates the correlation between husband and wife fixed effects (which includes education) to be 0.572, and estimates the correlation of persistent shocks to be 0.154 over the 1980-1985 period in a sample of married households. Attanasio et al. (2008) use Hyslop’s estimate for the correlation of shocks within the household, and thus choose a value very similar to ours.
4.4 Education and matching

We impose that the cohort- and gender-specific distributions \( F_t^g \) for the utility cost of attending college are log-normal, \( \ln \kappa \sim N(\kappa_t^g, \psi_t^g) \). The (constant) variances \( \psi_t^g \) are set so as to match changes in enrollment rates by gender between the initial and final steady states, assuming the same mean costs apply in both steady states. Intuitively, the variances regulate the gender-specific elasticities of enrollment rates to increase in the college wage premium. The fact that college graduation has increased more for women than for men (recall panel (C) in Figure 1) implies less dispersion in the distribution of female enrollment costs relative to that for men (see Table 1). Simultaneously, we set the sequences for cohort-specific means \( \kappa_t^g \) to match the level of college completion year by year.\(^{21}\)

To calibrate the matching probabilities, we use a simple strategy. The correlation coefficient \( \varrho^* \) between the education levels of husband and wife is set to 0.517, as explained above. Given values for \( \varrho^* \) and for the model’s equilibrium enrollment rates \( q_t^g \), equation (7) identifies the conditional probability \( \pi_{t}^{m}(h, h) \). The remaining matching probabilities follow from the constraints (5) and (6). The observed rise in educational attainment implies substantial changes in the matching probabilities. For example, across steady states \( \pi_{t}^{m}(h, h) \) rises from 0.43 to 0.79.

4.5 Preferences, debt limits, and pensions

The period utility function for a household is

\[
 u(c, n^m, n^f) = \frac{e^{1-\gamma}}{1-\gamma} + \psi (1 - n^m)^{1-\sigma} + \psi (1 - n^f)^{1-\sigma}. \tag{13}
\]

In this specification, there is no asymmetry in preferences between male and female time, so any differences by gender in the equilibrium distributions for hours worked will be driven by the gender wage gap. Note also that this preference specification allows for labor supply adjustments along both the intensive and extensive margins: if the wages of two spouses are sufficiently different, the lower-wage spouse will choose to supply zero market hours.

Estimates of relative risk aversion between one and two are common (see Attanasio, 1999, for a survey), so we set \( \gamma = 1.5 \). We set the utility weight of nonmarket time relative to

\(^{21}\)The empirical counterpart for the initial steady state is the fraction of 25- to 54-year-olds who were college graduates in 1967: 15.3% for men and 8.5% for women. The empirical counterpart for the final steady-state is an estimate of the fraction of college graduate 25-year-olds in 2002: 25.6% for men and 31.7% for women. In every year between the initial and final steady states, we target the graduation rates by gender for ages 25-29 plotted in panel (C) of Figure 1.
market consumption to \( \psi = 0.335 \) to match average household hours worked in the market, estimated to be 30% of the time endowment (assumed to be \( 15 \times 365 = 5,475 \) hours per year per individual) over the sample period.

The curvature parameter \( \sigma \) serves two purposes. First, the intertemporal elasticity of substitution for individual nonmarket time is given by \( 1/\sigma \), so \( \sigma \) regulates the Frisch elasticity of labor supply. Second, \( 1/\sigma \) is the static elasticity of substitution between male and female leisure. Consequently, \( \sigma \) will determine the allocation of time within the household. In particular, when leisure is interior for both spouses, relative leisure is given by

\[
\ln \left( \frac{1 - n_f}{1 - n_m} \right) = \frac{1}{\sigma} \ln \left( \frac{w_m}{w_f} \right).
\]

(14)

Thus, the extent to which within-household wage differentials translate into differences in market hours is increasing in \( 1/\sigma \).

We set \( \sigma = 3 \). This value satisfies three criteria. First, the implied mean Frisch elasticity of labor supply for men is 0.48 and the one for women is 1.46.\(^{22}\) These numbers are well within the range of gender-specific micro estimates (see Blundell and MaCurdy, 1999, for a survey of micro estimates, and Domeij and Flodén, 2006, for an argument based on liquidity constraints for why micro estimates may be downward biased). Second, this value exactly replicates the empirical ratio of average female to average male hours of 0.48 (averaged over the entire period). Third, with this choice the model replicates the empirical correlation of \(-0.11\) between year-on-year growth in husband’s wages and corresponding growth in wife’s hours.\(^{23}\) Satisfying these three criteria is an important indicator of the model’s ability to capture household behavior. The first and second results show that one can account for gender differences in average hours and in the sensitivity of hours to changes in wages without appealing to asymmetries between men and women in terms of how hours enter preferences, or in the process for individual wage shocks. The third result provides an implicit empirical validation for the degree of within-household risk-sharing that the model delivers through the joint labor supply decision. We conclude that this simple two-parameter \((\sigma, \psi)\) model of nonmarket work can account surprisingly well for the salient features of time allocation within the household.

\(^{22}\)Recall that the Frisch elasticity of labor supply is \((1/\sigma) (1 - n^\theta)/n^\theta\), so it is a function of hours worked. As female hours worked rise, the average elasticity for women in the model declines from 1.77 in 1967 to 1.25 in 2005. This fall is consistent with the findings of Blau and Kahn (2007), who document a decline in married women’s labor supply elasticities between 1980 and 2000.

\(^{23}\)The raw correlation over the sample period is \(-0.087\), and when correcting for measurement error the correlation falls to \(-0.11\). The correction assumes that hourly wages inherit all measurement error from hours, and that the variance of these errors is 0.02, as estimated by French (2004).
Following Storesletten, Telmer, and Yaron (2004), the discount factor $\beta$ is set so that agents have a realistic amount of wealth, and can thereby achieve an appropriate amount of self-insurance through savings. We set $\beta = 0.969$ to replicate the ratio of average wealth to average pre-tax earnings in 1992, which was 3.94 according to the 1992 Survey of Consumer Finances (SCF). This value for $\beta$ implies that the model economy has, on average, a small negative net foreign asset position (in 1992 foreign-owned assets are 12.1% of the domestic capital stock).

The ad-hoc borrowing constraint $a$ is calibrated to match the proportion of agents with negative or zero wealth. In 1983, this number was 15.5% (Table 1 in Wolff, 2000). The implied borrowing limit is 20% of mean annual individual after-tax earnings in the initial steady state.

The US social security system pays old-age pension based on a concave function of average lifetime earnings. Several authors have documented that the implied risk-sharing properties of the system are substantial (e.g., Storesletten et al., 2004). Including exactly such a system in our model would be computationally expensive, since two indexes of accumulated earnings would have to be added as state variables. Here, we adopt a simpler version capturing the amount of redistribution embedded in the US system: all workers receive the same lump-sum pension, $b$, the value of which is such that the dispersion of discounted lifetime earnings plus pension income in the final steady state of our economy is the same as in an alternative economy featuring the actual US Old-Age Insurance system. The implied value for $b$ is 24.5% of mean individual pre-tax earnings in the initial steady state (see Section A3 in the Appendix for details).

5 Macroeconomic implications

The purpose of this section is to investigate, through the lens of our calibrated model, the implications of changes in the wage structure for the evolution of the cross-sectional distribution over hours, earnings, and consumption. We therefore simulate the calibrated benchmark economy, in which all elements of the vector $\lambda_t$ are time-varying, and compare the model-implied paths for the cross-sectional moments of interest to their empirical counterparts computed from the CPS (for wages, hours, and earnings) and from the CEX (for consumption). We also conduct a set

\[\text{In comparing average household wealth across model and data, we exclude the wealth-richest 1% of households in the data, since the very richest households in the SCF are missing in both the model and in the CEX, PSID, and CPS (see Heathcote et al., 2010, for more discussion).}\]

\[\text{Recall that to estimate the time-varying parameters } \{\lambda_t\} \text{ we used data from the PSID, since our identification scheme relies on the panel dimension. We chose to use CPS data for the model evaluation because the CPS}\]
of decomposition experiments in which we change the components of \( \lambda \), one at a time, holding the other components fixed at initial steady-state levels.\(^{26}\) This approach allows us to assess the extent to which the predicted dynamics are primarily attributable to (i) skill-biased demand shifts, (ii) gender-biased demand shifts, (iii) changes in the variance of persistent shocks, and (iv) changes in the variance of transitory shocks. We focus on changes over time by demeaning all variances and correlations.\(^{27}\)

### 5.1 Labor supply

We first note that the model, estimated on data from the PSID, generates time series for male and female wage dispersion that are consistent with their empirical counterparts in CPS data. For men, this result is not too surprising, given that the statistical process for individual labor productivity is estimated on a sample of men, as is the college wage premium. The finding of close alignment between the model and data series for female wage dispersion provides ex-post support for assuming symmetric processes for male and female wages.

According to the model, changes in the wage structure have had a big impact on the distribution of hours, especially for women. Panel (A) of Figure 4 plots average female hours worked relative to average male hours. The model accounts for roughly three quarters of the increase in relative female hours over this period, and essentially the entire rise after 1980. The decomposition into the four elements of \( \lambda \) (panel D) shows that this increase is entirely driven by a narrowing wage gap, i.e., the gender-biased demand shift component \( \lambda^G_t \) (see also Jones, Manuelli, and McGrattan, 2003). Because the model accounts for the bulk of the observed increase in female hours, it can be used to address the implications of the transition from the traditional single-male-earner household toward the current dual-earner prototype. At the same time, the fact that our model falls short of replicating the increase in female hours in the 1960s and early 1970s suggests a role for alternative explanations during this period, such as cultural change (Goldin, 2006; Fernández and Fogli, 2009), rapid technological progress in the home

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\(^{26}\)For each decomposition, we compute a new path for \( Z_t \) following the strategy described in Section 4.

\(^{27}\)Properly comparing levels of inequality across model and data would require a careful treatment of measurement error and preference heterogeneity. However, one can safely compare trends in inequality as long as the variances of measurement error and preference heterogeneity are stationary (see Heathcote et al., 2008b).
sector (Greenwood, Seshadri, and Yorukoglu, 2005), or declines in child-care costs (Attanasio et al., 2008).  

Figure 4 also documents the model’s predictions for hours dispersion within groups of male and female workers. As in the data, the model generates more dispersion in female hours than in male hours: the average variances of log hours are 0.23 and 0.02, respectively. The reason is that the Frisch elasticity for market hours is decreasing in average hours worked, and the gender wage gap means that women typically work less than men. In terms of trends, panel (B) shows that the variance of log male hours in the model is essentially flat, as in the data, except for a small rise toward the end of the sample due to increasingly volatile productivity. The model also predicts a flat time profile for the variance of female hours, in contrast to the observed decline in the data (panel C). The model profile reflects the existence of several offsetting forces (panel F). Larger transitory and persistent shocks drive up dispersion in female hours. At the same time, the narrowing gender gap increases average female hours, thereby reducing the average Frisch elasticity for female labor supply and hours variability. Note that if the gender wage gap were to vanish entirely in our symmetric model, the distribution for female market hours would become identical to that for males.  

Figure 5 plots the cross-sectional correlation between the individual log wage and individual log hours. As documented in Section 2, there is a large rise in the wage-hour correlation for men in the 1970s and 1980s. The model reproduces both the magnitude and the timing of this increase. Panel (C) of the figure indicates that each component of the wage process is

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28 We also find evidence of positive selection in the model. The gender gap for average observed wages is smaller than for offered wages, because low-wage women married to high-wage men tend to not work full time. Over time, as increasing female wages induce less productive women to work, this selection effect weakens in the model. The fact that the gap in offered wages narrows rapidly (relative to the gap in observed wages) helps explain why the model generates such a surge in female market work. Blau and Kahn (2006) provide empirical support for this type of selection in the United States in the 1980s and 1990s, using a wage imputation procedure for women working few or zero hours.  

29 A closer examination of the CPS data indicates that, mechanically, the main reason for the decline in women’s hours dispersion is the increased clustering at full-time work (i.e., 2,000 hours per year). This decline could be artificially inflated by heaping (i.e., rounding off) in hours reports, a typical bias of retrospective surveys. One way to reproduce this trend would be to allow for nonconvexities in labor supply in the model.  

30 The average level of this correlation is positive in the model but negative in the data. In large part, the low number in the data reflects measurement error (the “division bias”): if an individual’s report of hours worked is too high (low), their imputed hourly wage, computed as earnings divided by hours, is automatically too low (high). The CPS offers two alternative ways to estimate hours worked, based on two different questions, one about usual weekly hours worked this year, and the other about hours worked last week. The first question should provide a more accurate estimate for total hours worked in the previous year, but it was only asked beginning with the 1976 survey. Because we want to measure hours in a consistent way across our entire sample period, we use the first question. However, for the post-1976 period we computed moments both ways. Reassuringly, the
important for determining the overall evolution of the male wage-hour correlation. Given our assumption on risk aversion ($\gamma > 1$), wealth effects cause individual hours to move inversely with uninsurable wage changes, whereas market hours will move in step with wage changes that can be insured either through saving or through intra-household time reallocation. In the context of our model, the secular upward trend in the college premium has been largely uninsurable (conditional on educational choice) and has reduced the wage-hour correlation. However, this effect is more than offset by the positive impact of more volatile transitory shocks – which are easy to insure through precautionary savings – and by the effect of gender-biased demand shifts. Labor demand shifts toward women drive up the correlation between male hours and male wages because the larger is the fraction of household income attributable to the female, the smaller is the impact of a shock to the male wage on household consumption, and thus the smaller the wealth effect on male hours.

The path for the female wage-hour correlation (panel B) is flatter than the correlation for men, in both the model and the data. As women’s share of household earnings has risen, household consumption has responded increasingly strongly to female wage shocks, and these larger wealth effects moderate the increase in the wage-hour correlation. This also explains why the wage-hour correlation for women is higher than for men, in both the model and the data: on average the wealth effects associated with wage changes are smaller for women.

Finally, we note that the variance of individual earnings predicted by the model (not plotted) lines up closely with the data for both men and women. In particular, in both model and data, the increase in male earnings inequality is larger than the increase in wage inequality, which mathematically reflects an increasing wage-hour correlation.

## 5.2 Household earnings and consumption

Figure 6 shows the time paths for the variances of household earnings and household consumption. The variance of log household earnings is one moment for which the CPS and the PSID are not in full agreement, particularly toward the end of the sample, where inequality rises more
rapidly in the CPS.\textsuperscript{31} The increase in household earnings inequality generated by the model (14 log points) lies in between the CPS and PSID series and is closer to the PSID, as might be expected given that we use the PSID to estimate the wage process. The rise in household earnings inequality in our CEX sample also lies in between the corresponding increases in the CPS and PSID.

Panel (C) shows that the dynamics of household earnings dispersion are mainly driven by increases in the variances of transitory and persistent shocks. The model-generated rise in household earnings inequality is smaller than the rise in individual earnings inequality because wage shocks are correlated within the household, and household earnings can be further smoothed by reallocating hours between husband and wife. In addition, demand shifts in favor of women boost female hours, increasing the scope for within-household insurance, and further mitigating the rise in household earnings inequality. The inequality-increasing role of skill-biased demand shifts is muted by the imperfect correlation of education within the household, and by the negative effect of these shifts on the wage-hour correlation (recall Figure 5).

Panel (B) describes the dynamics of the variance of household log consumption. CEX data show a modest increase in consumption inequality since 1980. The increase in consumption inequality generated by the model is similar to that observed in the data.

The counterfactual experiments in which only one component of the wage process is time-varying shed light on the mapping from earnings inequality to consumption inequality. A comparison of panels (C) and (D) reveals that demand shifts have quantitatively similar impacts on earnings inequality and consumption inequality. Similarly, Attanasio and Davis (1996) find that low-frequency changes in relative wages between educational groups lead to similar changes in relative consumption.\textsuperscript{32}

In contrast to demand shifts, changes in the variance of wage risk have very different effects on earnings and consumption inequality, reflecting self-insurance through savings. When only the variance of transitory shocks is time-varying, the increase in the variance of consumption over the 1965-2003 period is just 9% of the increase in the variance of household log earnings. This confirms that transitory shocks can be smoothed effectively with the risk-free asset. Isolating the impact of increasingly volatile persistent shocks delivers a rise in consumption inequality

\textsuperscript{31}We discuss the sources of this discrepancy in Section A1 of the Appendix. See also Heathcote et al. (2010).

\textsuperscript{32}Demand shifts in favor of educated labor induce a change in consumption inequality even though they are assumed to be foreseen (after 1965). This is because high school graduates who enter the economy after the rise in the college premium cannot avoid low permanent income and consumption levels.
61% as large as the rise in earnings inequality. Thus, households in the model achieve a degree of self-insurance even against highly persistent shocks, a finding consistent with previous work by Blundell, Pistaferri, and Preston (2008), and Kaplan and Violante (forthcoming). In the benchmark simulation, when all dimensions of the wage structure are time-varying, the increase in consumption inequality is 40% as large as the increase in earnings inequality.\footnote{Over the period of our CEX sample, 1980-2003, this ratio is 48% in the model and 54% in CEX data.}

Krueger and Perri (2006, Figures 2 and 5) decompose the rise in consumption inequality into changes within and between groups. They document that half of the rise in consumption inequality was due to residual (within-group) inequality. They conclude that the amount of consumption insurance available to US households exceeds that available in a standard bond economy model (e.g., Huggett, 1993). Our model, which has more channels of self-insurance than Huggett’s, generates an increase in within-education-group consumption inequality that is precisely half of the total. However, in the data the rise of the within-group component occurs mostly in the 1980s, whereas in our model it grows smoothly throughout the 1990s as well. One possible interpretation of this finding is that households’ borrowing constraints were relaxed in the 1990s, which is the main argument of Krueger and Perri (2006).

Finally, although the focus of our analysis is on changes in cross-sectional inequality over time, we have also explored the predictions of the model along the life cycle dimension. For example, the model closely replicates the evolution of means and variances for household earnings and consumption when following the 1980 cohort over time (see Section A4 in the Appendix for details).

### 6 Welfare implications

The finding that our structural model broadly reproduces empirical trends in the joint distributions for hours, earnings and consumption indicates that widening wage inequality is the key factor underlying rising economic inequality among US households. In this light, we apply the model to assess the welfare implications of the estimated changes in the wage structure.

#### 6.1 Methodology

Welfare calculations must factor in education costs, because rising enrollment implies both higher wages and higher utility costs for attending college. We choose to measure welfare gains
or losses by computing the percentage change in lifetime consumption required to give agents facing the steady-state education cost distribution the same average lifetime utility in the initial steady state as they enjoy when entering the labor market in any subsequent year $t$. Holding fixed the education cost distribution in the welfare calculations has three advantages. First, welfare can be compared over time, because welfare is always evaluated from the perspective of the same set of agents. Second, we can isolate the welfare effects of changes in the wage structure (as opposed to changes in education costs), which is our main focus. Third, welfare can be compared across alternative models for expectations (perfect foresight versus myopic beliefs), since the steady-state distribution for utility costs is the same in both cases.

More specifically, the equivalent-variation welfare gain for households of type $(e^m, e^f)$ from entering the economy in year $t$ rather than in the initial steady state (denoted by the subscript $*$) is the value $\phi_t$ that solves

$$
2 E_t \left\{ \sum_{j=0}^{J-1} \beta_j^j \zeta u \left( c_{t+j}, n_{t+j}^m, n_{t+j}^f \right) \left| e^m, e^f \right\} - \sum_{g \in \{m,f\}} I_{\{e^g=h\}} E_* \left[ \kappa | \kappa \leq \hat{\kappa}_t^g \right] \right\}
$$

where $\zeta_j$ are unconditional survival probabilities, \left\{ c_{s_j}, n_{s_j}^m, n_{s_j}^f \right\}_{j=1}^{J-1}$ are equilibrium allocations for households facing the steady-state wage structure $\lambda_*$, \left\{ c_{t+j}, n_{t+j}^m, n_{t+j}^f \right\}_{j=1}^{J-1}$ are equilibrium allocations for households entering the economy at date $t$ and facing the wage structure $\{\lambda_{t+j}\}_{j=1}^{J-1}$, and where $E_t$ and $E_*$ denote ex post averages for agents entering the labor market in year $t$ and in the initial steady state. Recall that the average education cost paid by college graduates of gender $g$ is the expected value of $\kappa$ conditional on $\kappa$ being less than the threshold below which college is the optimal education choice: $\hat{\kappa}_t^g$ denotes this threshold when entering the economy in period $t$ and $\hat{\kappa}_*^g$ when entering in the initial steady state. Again, the expectations over education costs are taken with respect to the steady-state distributions $F_*^g$.

The average welfare gain across all household types is defined by a similar equation, except that both sides of (15) now involve weighted averages across different household types, with the weights determined by enrollment rates $F_*^g (\hat{\kappa}_*^g)$ and $F_*^g (\hat{\kappa}_*^g)$, respectively.\[34\]

\[34\]Recall that, in our simulations, tax rates are assumed to be constant over time, as is the real value of the public pension for retirees. The government balances its budget in each period. Thus any shifts over time in average earnings or wealth translate into changes in tax revenue which, in turn, generate changes in
### 6.2 Results under perfect foresight

Panel (A) of Figure 7 plots the average welfare effect of changes in the wage structure. Overall, these changes have been welfare improving: relative to the initial steady state, the average gain from entering the economy in 2000 is equivalent to a 3.1% increase in consumption. However, welfare gains are non-monotone: for example, entering the economy in the late 1970s implies smaller gains relative to entering in the early 1960s.

Panel (C) of Figure 7 plots the contribution of each component of structural change (persistent and transitory shocks, skill-biased and gender-biased demand shifts) to the overall welfare effect. Larger transitory shocks translate into welfare gains of around 0.3% from the mid-1960s and onward. These shocks are easily insurable and offer opportunities for efficient reallocation of hours worked, which in turn increases labor productivity and, hence, welfare. The large increase in the variance of persistent shocks is the main source of welfare losses for the typical US household. Since these shocks are so durable, buffer-stock savings are of limited use for consumption smoothing. Thus rising residual wage variability (transitory plus persistent shocks) generates large welfare losses, rising toward 4% of consumption for entry after 2000.

Stronger relative demand for female labor reduces average labor productivity, because it increases hours worked by women who earn less than men, on average. At the same time, households opt for a more even allocation of time within the household, which increases average utility from leisure and facilitates within-household sharing of labor market risk. As expected, the positive effects dominate.\(^{35}\) For example, the gain from the gender-biased demand shifts is 1.4% of consumption for year 2000.

Panel (C) also shows that skill-biased demand shifts generate large welfare gains. Recall that both demand shifts favoring graduates (an increase in \(\lambda_{t}^{\delta}\)) and bigger persistent shocks (an increase in \(\lambda_{t}^{\omega}\)) imply increased cross-sectional consumption dispersion (see Figure 6, panel D). However, the two trends have opposite implications for welfare. In response to the skill-biased demand shift, individuals have the opportunity to avoid the low-wage outcome through a behavioral response: infra-marginal agents change their education decision, relative to the non-valued government consumption. We find that model government consumption rises from 18.8 percent of average household pre-tax income in the initial steady state to 19.9 percent in 2000. If we had assumed constant government spending as a share of income, and balanced the government budget by reducing tax rates over time, we would have found larger welfare gains than those reported below.

\(^{35}\)Gender-biased demand shifts benefit all households in the model, because we focus on married couples. Single men would lose from the growth in the relative demand for female labor.
initial steady state, in favor of college. The rise in college education witnessed in the US (and replicated in the model) indicates that many households took advantage of this opportunity.

Overall, the welfare gains documented in panel (A) are driven by a combination of behavioral responses to gender-biased and skill-biased demand shifts. Gender-biased demand shifts are relatively more important for the welfare of entrants in the 1960s and 1970s, while skill-biased demand shifts dominate in the 1990s and 2000s. This pattern reflects the fact that the narrowing of the gender wage gap was concentrated in the earlier period, while growth in the skill premium was a phenomenon of the 1980s and 1990s.

Panel (B) shows welfare changes conditional on household type. There are dramatic changes in the pattern of relative gains and losses over time. Early on, all households in which the husband is a college graduate lose, while high school graduate households gain. The reason is that early labor market entry means working through the unexpectedly low skill premium of the 1970s. In later years, skill-biased demand shifts generate very large welfare gains for college-educated households experience, and large welfare losses for high school households: these losses reach 2% of consumption in the late 1990s. Welfare losses for households in which neither spouse has a college degree are particularly noteworthy because this group always constitutes a majority of the population.

The average welfare gain in panel (A) is a weighted average of the gains for the different types reported in panel (B), with the weights varying over time as enrollment rates adjust. In the early years the average gain reflects gains for low-skilled households, who constitute the vast majority. The weight on households with at least one college-graduate spouse rises in successive periods, with positive and sizeable implications for average welfare.36

Finally, we find large heterogeneity in ex-post welfare effects within education groups which are driven by differences in the histories of persistent and transitory shocks. For example, 44% of high school households in 2000 experience a welfare gain ex post, not withstanding the 1.9% average welfare loss for this group. The 90-10 differential in the distribution of ex-post welfare gains for this household type is 20.9%.

36Note that unconditional average welfare falls in 1965 (panel A), whereas welfare for households with at least one college graduate increases (panel B). Anticipating a lower college premium in the 1970s, the education cost threshold for college attendance is lower for (informed) agents in 1965 than for (uninformed) agents in 1964. This translates into higher average ex post welfare for college-educated households in 1965, since welfare is measured net of (now smaller) education costs. At the same time, average welfare for 1965 declines because lower enrollment means foregoing positive externalities: the costs of education are borne at the individual level, while the gains in terms of higher wages and consumption are enjoyed by both spouses.
6.3 Insurance and opportunities

In interpreting our welfare results, we pointed to three sources of welfare gain: (i) gains from switching from high school to college when the relative price of college-educated labor rises; (ii) gains from reallocating time within the household when the gender wage gap narrows; and (iii) gains from concentrating labor effort in periods of high productivity while using savings to smooth consumption when residual wage volatility increases.

We now quantify the relative importance of these three margins of adjustment to structural change (college enrollment, hours worked, and savings) by shutting them down, one at a time. In the first experiment, we hold fixed enrollment rates at each date at their initial steady-state levels. In the second, we assume that at each date, male and female hours in each household are equal to their population averages in the initial steady state. In the third, we assume that each household must set consumption equal to income in every period. In each of these counterfactual experiments, all preference and technology parameters are exactly equal, at each date, to their values in our baseline model.37

The welfare findings from these experiments (panel D of Figure 7) are revealing. The heterogeneous-agent incomplete-markets literature has focused on self-insurance via saving and, to a lesser extent, labor supply as the key margins of adjustment to labor market risk. Indeed, we find that these margins are important, and, as it turns out, roughly equally so. When either of these margins is shut down, large average welfare gains turn into welfare losses. Labor supply is a critical margin, primarily because it allows women to increase participation in the face of a narrowing gender gap. Savings is critical in buffering larger idiosyncratic shocks, and for smoothing consumption over the life cycle in the face of trends in the prices of specific types of labor. However, the margin of adjustment that matters the most for welfare is the education choice. Had agents been unable to increase enrollment in response to a widening skill premium, they would have suffered enormous welfare losses, surging above 10% of consumption in the 1990s. The intuition is twofold: (i) absent growth in enrollment there would have been much less growth in average earnings, and (ii) skill-biased demand shifts would have generated more growth in the college wage premium, translating into greater inequality.

37 However, because in these experiments agents make different choices (optimally adjusting along the margins that remain open to compensate for the particular margin that is shut down) these economies feature different, counterfactual sequences for equilibrium prices.
6.4 Comparison with earlier welfare calculations

Attanasio and Davis (1996), Krueger and Perri (2004), and Storesletten (2004) compute welfare effects of changes in inequality directly from the empirical CEX distributions of consumption and leisure. They find losses of 1% to 2%. Their approach has the advantage that no assumptions have to be made on markets, technologies, or agents’ choice sets. However, it has two severe drawbacks relative to our structural approach, which lead these authors to overestimate welfare losses.

First, absent a structural model, one cannot predict the effect of changes in relative wages on average output, consumption and hours. Indeed, the empirical approach ignores level effects by detrending the data. In contrast, our structural approach does incorporate level effects since the wage structure influences aggregate output: skill- and gender-biased demand shifts influence output through their effects on human-capital accumulation and female participation, while rising wage volatility impacts productivity through modified labor supply decisions.

Our structural approach instead incorporates level effects since the wage structure influences aggregate output through three channels: skill- and gender-biased demand shifts influence output through their effects on human-capital accumulation and female participation, and a rise in transitory wage volatility leads to higher productivity through modified labor supply decisions.

Second, when computing welfare effects, Attanasio and Davis (1996) take averages over education groups, holding the weights on the two groups fixed. This approach exaggerates welfare losses because it ignores the fact that infra-marginal agents can choose to switch from the low- to the high-education group when the college premium rises.\(^{38}\) The wage gains and switching costs of agents who exercise this option do appear in our welfare calculation in equation (15).

\(^{38}\)To illustrate this, consider an economy with two groups, low-skilled and high-skilled. Suppose the difference in consumption between the groups increases between \(t\) and \(t + 1\), while average consumption weighted by the date \(t\) population shares remains constant. This would reduce average welfare. However, if low-skilled agents could become high-skilled by paying a cost, then the appropriate welfare comparison would weight the groups differently at \(t\) and \(t + 1\). Since switching is optimal for the switchers, this latter welfare calculation will imply a smaller welfare loss.
6.5 Myopic beliefs about the wage structure

Up to this point we have assumed that from 1965 onward, agents perfectly foresee the sequences of future risks and prices, \( \{\lambda_t^r, \lambda_t^\omega, p_t^e\}_{t=1965}^\infty \). We now consider an alternative model, where at each date \( t \) agents myopically expect that the date \( t \) wage structure \( \{\lambda_t^r, \lambda_t^\omega, p_t^g\} \) will remain unchanged at all future dates. Each period agents are surprised when prices and shock variances do in fact change. Myopic beliefs and perfect foresight represent polar extreme models for expectations, and presumably the truth lies somewhere in between the two.

We calibrate the myopic beliefs model following the same strategy as the perfect foresight economy. The steady states of the two models are identical, since myopia and perfect foresight coincide when the wage structure is time invariant. However, the sequences for skill and gender-biased demand shifts \( \{\lambda_t^S, \lambda_t^G\} \) and the sequences for the means of the education cost distribution \( \{\pi_t^C\} \) are recalibrated such that the myopic model replicates the observed time series for the college premium, the gender wage gap, and the college completion rates by gender, respectively. The main change relative to the perfect foresight case is lower average education costs in the 1970s and 1980s. Lower costs are required to support high observed enrollment levels for cohorts who mistakenly believe that the college premium will remain permanently low.

The positive implications of myopic beliefs, in terms of the evolution of the cross-sectional distributions of hours, earnings, and consumption, are remarkably similar to those of the perfect foresight model.\(^{39}\) This finding is intuitive, since both economies, by construction, generate the same relative price series and the same enrollment rates by gender (and thus the same household distribution by education composition). Time allocation between work and leisure within the household is determined by static comparative advantage, i.e., by current relative prices of labor services, which are the same in both economies. Consumption and savings behavior is affected by differences in expectations about the future, and consumption inequality grows slightly more under myopic beliefs, indicating less inter-temporal smoothing. Overall, however, we conclude that the model’s ability to explain trends in inequality is not materially influenced by how much foresight agents have about the future wage structure.

In contrast, beliefs do matter for the welfare effects of the changing wage structure. An important theme of our analysis is that individuals take advantage of the opportunities offered

\(^{39}\)Section A5 of the Appendix reports a pairwise comparison of the time paths for the relevant moments for the two economies. Visually, the moments are virtually indistinguishable across the perfect foresight and myopic beliefs models. We also plot the sequences \( \{\tilde{\pi}_t^C\} \) in both economies.
by the changing college and gender wage gaps. The more accurately agents forecast future prices, the more effectively they can exploit these opportunities. By comparing welfare effects across the perfect foresight and myopic beliefs models, we can measure the value of knowledge about the future aggregate state of the economy.

The welfare effects of changes in the wage structure are still defined by equation (15) under myopic beliefs. However, while in the perfect foresight model ex-ante expected and ex-post average lifetime utilities coincide after 1965, in the myopic model they do not. Our consumption-equivalent welfare measures are based on average realized utility, but recognize that agents make decisions, including enrollment decisions, based on expected utility calculations.

Figure 8 shows that myopic beliefs imply (average) welfare losses of up to 1.2% in the late 1970s, compared to welfare gains in excess of 1% in the perfect foresight economy. Welfare losses are larger under myopic beliefs in the 1970s and 1980s because more agents choose not to go to college under the mistaken belief that the college premium will remain low in the future. As the economy converges to the final steady state, which is identical under both models for expectations, the gap between the two series for welfare gains narrows. Panel (B) breaks down welfare effects by household type. High-school-educated households experience similar welfare losses to their counterparts in the perfect foresight economy. However, college-educated households who enter in the 1970s and 1980s experience much larger average gains under myopic beliefs. The reason is that, during this period, individuals do not anticipate the subsequent rise in the college premium, and thus the average college attendee has a lower utility cost for attending college than under perfect foresight. Note that even though welfare conditional on household type is larger under myopic beliefs than under perfect foresight, the unconditional average welfare gain, weighted by household type, is smaller. This pattern underscores the conclusion that average welfare losses under myopic beliefs are driven by short-sighted agents failing to exploit potential welfare gains from college attendance.

7 Conclusions

Since the early 1970s, the US economy has experienced a structural change in the wage distribution along several dimensions: the college premium doubled, the gender gap halved, and wage

\[\text{Note that before 1969, welfare is slightly higher under myopic beliefs than under perfect foresight. The reason is that during this period, myopic individuals have higher enrollment rates, since they do not foresee the fall in premium after 1970. Higher enrollment then translates into higher welfare through the education externality, as explained above.}\]
variability increased substantially. In this paper, we studied the macroeconomic and welfare implications of all these changes through the lens of a version of the neoclassical growth model with incomplete markets and overlapping generations. Our model extends the prototypical framework by adding an education choice, a model of the family in which husbands and wives face imperfectly correlated shocks to wages, and a production technology where labor inputs are differentiated by gender and education.

Our first novel result is that the model, with the changing wage structure as the key input, accounts for the salient trends in the inequality of labor supply and consumption in the United States. Our study abstracted from other competing forces, such as the rise in the share of single households, changes in the progressivity of the tax code, and developments in financial markets. While these forces likely played some role, our analysis establishes that the transformation of the wage structure is the predominant force behind the evolution of the distribution of hours and consumption, and hence welfare, across US households.

Our second novel, and somewhat surprising, result concerns welfare. When we apply our structural model to quantify the welfare consequences of the observed changes in wage structure, we find that these changes have been welfare-improving. Notably, on average, a household facing the 1960s wage structure would be willing to pay over 3% of lifetime consumption in order to face, instead, the wage structure of the 2000s. Welfare gains are driven by individuals' ability to respond to structural change by adjusting savings, labor supply, and education choices. Two important caveats are in order. First, the size of welfare gains hinges on the degree of foresight individuals have about the evolution of the skill premium. Second, average welfare gains mask large differences by household type: high-school-educated households are hit very harshly by the adverse demand shift after 1980. Thus, under a max-min Rawlsian welfare function, the new wage structure would imply welfare losses, since the poorest households become even poorer.

The sharp rise in US economic inequality has featured prominently in the public policy debate. Contrary to common belief, our study suggests that policies that offset the rise in earnings inequality would not necessarily be welfare improving, since we find the skill-biased demand shift to have produced welfare gains. For example, more progressive taxation dissuades individuals from acquiring additional education in response to a widening skill premium. Offering additional insurance against increasingly volatile persistent shocks would be a more effective policy. However, in practice, welfare-augmenting and welfare-reducing drivers of inequality cannot be easily unbundled. The policy challenge is to design institutions and tax-transfer
schemes that deliver insurance against misfortune at birth and later in life, while preserving incentives for agents to make efficient human capital investment and labor supply decisions.
References


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Figure 1: Cross-sectional facts: model inputs. All time series are demeaned and means are reported in brackets within the legends. Source: CPS 1967-2005. Sample: Married households where the husband is 25-59 years old. See Section 2 of the text and Section A1 of the Appendix for details on the selection and construction of the sample.
Figure 2: Cross-sectional facts: model targets All time series are demeaned and means are reported in brackets within the legends. Source: CPS 1967-2005 and CEX 1980-2003. Sample: Married households where the husband is 25-59 years old. See Section 2 of the text and Section A1 of the Appendix for details on the selection and construction of the sample.
Figure 3: Panels (A) and (B): Variances of persistent and transitory wage shocks estimated from the PSID 1967-2000. Each panel reports point estimates for the variances and bootstrapped standard errors based on 500 replications. See Section A2 of the Appendix for details. Panels (C) and (D): Results of the internal calibration for skill- and gender-biased demand shifts. The paths for these two variables allow the model to replicate the empirical college wage premium and gender wage gap reported in panels (B) and (D) of Figure 1. See Section 4.3 for details. This figure displays all four components of the $\{\lambda_t\}$ sequence.
Figure 4: Model-data comparison and decomposition. Panels (A), (B) and (C) plot the female-male hours ratio and of the dispersion in log hours worked for males and females. Both model and data series are demeaned and means are reported in brackets within the legends. Panels (D), (E) and (F) plot the corresponding variable (the one in the above panel) in all four model counterfactuals when we let the components of \( \{ \lambda_t \} \) vary one at a time. The labels in the legend refer to the specific component turned on in the experiment. “Pers” denotes the variance of the persistent shock; “Trans” the variance of the transitory shock; “SB” skill-biased demand shifts, and “GB” gender-biased demand shifts.
Figure 5: Model-data comparison and decomposition. Panels (A) and (B) plot the correlations between log wages and log hours for men and women. Both model and data series are demeaned and means are reported in brackets within the legends. Panels (C) and (D) plot the corresponding variable (the one in the above panel) in all four model counterfactuals when we let the components of $\{\lambda_t\}$ vary one at a time. The labels in the legend refer to the specific component turned on in the experiment. “Pers” denotes the variance of the persistent shock; “Trans” the variance of the transitory shock; “SB” skill-biased demand shifts, and “GB” gender-biased demand shifts.
Figure 6: Model-data comparison and decomposition. Panels (A) and (B) plot the dispersion in log household earnings and log consumption. Both model and data series are demeaned and means are reported in brackets within the legends. Panels (C) and (D) plot the corresponding variable (the one in the above panel) in all four model counterfactuals when we let the components of $\{\lambda_t\}$ vary one at a time. The labels in the legend refer to the specific component turned on in the experiment. “Pers” denotes the variance of the persistent shock; “Trans” the variance of the transitory shock; “SB” skill-biased demand shifts, and “GB” gender-biased demand shifts.
Figure 7: Panel (A) plots the average welfare gain from the changing wage structure, cohort by cohort (see Section 6.1 for details on the calculation.) Panel (C) plots the average welfare gain in all four model counterfactuals when we let the components of \( \{ \lambda_t \} \) vary one at a time. The labels in the legend refer to the specific component turned on in the experiment. “Pers” denotes the variance of the persistent shock; “Trans” the variance of the transitory shock; “SB” skill-biased demand shifts, and “GB” gender-biased demand shifts. Panel (C) plots average welfare gain by household type. “Col-Col” denotes households where both spouses are college graduates; “Col-HS (HS-Col)” households where the husband (wife) has a college degree and the wife (husband) a high school degree; “HS-HS” households where both spouses are high school graduates. Panel (D) plots average welfare in the baseline and in the counterfactuals where agents’ choices are restricted. See Section 6.3 for details on the various experiments.
Figure 8: Panel (A) plots the average welfare gain from the changing wage structure, cohort by cohort in the model with perfect foresight and in the model with myopic beliefs. See Section 6.5 for a description of the latter economy. Panel (B) plots the average welfare gain by cohort in the economy with myopic beliefs by household type. “Col-Col” denotes households where both spouses are college graduates; “Col-HS (HS-Col)” households where the husband (wife) has a college degree and the wife (husband) a high school degree; “HS-HS” households where both spouses are high school graduates.
This Appendix is organized as follows. Section A1 describes the household surveys used in the paper, outlines the sample selection criteria, and compares the time trends for various empirical moments across surveys. Section A2 discusses in detail the specification, identification, and estimation of our statistical model for individual wage dynamics. Section A3 illustrates the numerical algorithm designed to compute the equilibrium of the model under both perfect foresight and myopic beliefs. Section A4 compares the evolution of inequality over the life cycle in the model and in the data. Section A5 portrays the evolution of some key cross-sectional moments in the perfect foresight model and in the myopic beliefs model.

A1 Data description

Our sources for individual- and household-level data are the Panel Study of Income Dynamics (PSID), the Current Population Survey (CPS), and the Consumer Expenditure Survey (CEX). Since all three data sets are widely used for microeconometric, and more recently, for quantitative macroeconomic research, we shall only briefly describe them here.

**PSID:** The PSID is a longitudinal study of a representative sample of US individuals (men, women, and children) and the family units in which they reside. Approximately 5,000 households were interviewed in the first year of the survey, 1968. From 1968 to 1997, the PSID interviewed individuals from families in the sample every year, whether or not they were living in the same dwelling or with the same people. Adults have been followed as they have grown older, and children have been observed as they advance through childhood and into adulthood, forming family units of their own (the “split-offs”). This property makes the PSID an unbalanced panel. Since 1997, the PSID has been biennial. The most recent year available, at the time of our analysis, is 2003. In 2003, the sample includes over 7,000 families. The PSID
consists of various independent samples. We focus on the main and most commonly used, the so-called SRC sample, which does not require weights, since it is representative of the US population. Questions referring to income and labor supply are retrospective, e.g., those asked in the 1990 survey refer to calendar year 1989.

**CPS:** The CPS is a monthly survey of about 50,000 households conducted by the Bureau of the Census for the Bureau of Labor Statistics. The sample is selected to represent the civilian noninstitutional population. Respondents are interviewed to obtain information about the employment status of each member of the household 16 years of age and older. The CPS is the primary source of information on the labor force characteristics of the US population. Survey questions cover employment, unemployment, earnings, hours of work, and other indicators. A variety of demographic characteristics is available, including age, sex, race, marital status, and educational attainment.

In our investigation, we use the Annual Social and Economic Supplement (so-called March Files) in the format arranged by Unicon Research. Computer data files are only available starting from 1968, and the latest year available, at the time of our research, was 2006. In all our calculations, we use weights. As for the PSID, questions referring to income and labor supply are retrospective.

**CEX:** The CEX is a survey collecting information on the buying habits of American consumers, including data on their expenditures, income, and consumer unit (household) characteristics. The data are collected by the Bureau of Labor Statistics and used primarily for revising the CPI. The data are collected in independent quarterly Interview and weekly Diary surveys of approximately 7,500 sample households (5,000 prior to 1999).

We use the data set constructed from the original CEX data by Krueger and Perri (2006) and available on the authors’ websites. As is common in most of the previous research, their data uses only the Interview survey, which covers around 95% of total expenditures. Frequently purchased items such as personal care products and housekeeping supplies are only reported in the Diary survey. The period covered by their data is 1980-2003. CEX data before 1980 are not comparable to the later years. Households who are classified as incomplete income respondents by the CEX and have not completed the full set of five interviews are excluded. We refer to Krueger and Perri (2006) for additional details on the data construction.

**Variable definitions:** The calibration of the model and its evaluation are based on a set
of cross-sectional first and second moments constructed from both the PSID and the CPS. The key variables of interest are: gross (i.e., before-tax) annual labor earnings, annual hours, hourly wages, and household consumption. We always construct hourly wages as annual earnings divided by annual hours worked. Nominal wages, earnings, and consumption are deflated with the CPI and expressed in 1992 dollars.

In the PSID, gross annual earnings are defined as the sum of several labor income components including wages and salaries, bonuses, commissions, overtime, tips, etc. Annual hours are defined as “annual hours worked for money on all jobs including overtime.”

In the CPS, gross annual earnings are defined as income from wages and salaries including pay for overtime, tips, and commissions. Annual hours worked are constructed as the product of weeks worked last year and hours worked last week. Until 1975, weeks worked are reported in intervals (0, 1-13, 14-26,..., 50-52). To recode weeks worked for 1968-1975, Unicon grouped the data in a few years after 1975 by intervals and computed within-interval means. These means from the later years were applied to the earlier years. The variable “hours worked last week at all jobs” is not ideal, but it is the only one continuously available since 1968 and comparable across years. Starting from the 1976 survey, the CPS contains a question on “usual weekly hours worked this year.” Even though levels differ, trends in mean hours, in their variance and in the wage-hour correlation, which are the focus of our study, are virtually equivalent across the two definitions.

In the CEX, gross annual earnings refer to the amount of wage and salary income before deductions received in the past 12 months. Since we noticed that in the Krueger-Perri file there were some missing values for earnings, we merged earnings data from the CEX Public Release Member files (provided to us by Orazio Attanasio) into the Krueger-Perri file and use the former observations whenever earnings data were missing in the original Krueger-Perri file. Annual hours worked are defined as the product of “number of weeks worked full or part time by member in last 12 months” and “number of hours usually worked per week by member.”

Our benchmark definition for consumption is the same as Krueger and Perri, i.e., the sum of expenditures on nondurables, services, and small durables (such as household equipment) plus imputed services from owned housing and vehicles. Each expenditure component is deflated by an expenditure-specific, quarter-specific CPI. Household expenditures are equivalized through the Census scale. We label this variable ND+. See Krueger and Perri (2006) for further details.
### Table A1: Sample Selection in PSID, CPS, and CEX

<table>
<thead>
<tr>
<th></th>
<th>PSID (67-96, 98, 00, 02)</th>
<th>CPS (67-05)</th>
<th>CEX (80-03)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#dropped</td>
<td># remain</td>
<td>#dropped</td>
</tr>
<tr>
<td>Initial sample (married househ.)</td>
<td>68,860</td>
<td>-</td>
<td>1,312,864</td>
</tr>
<tr>
<td>Age of husb. between 25-59</td>
<td>10,274</td>
<td>58,586</td>
<td>354,256</td>
</tr>
<tr>
<td>Hours worked of husb. ≥ 260</td>
<td>1,927</td>
<td>56,659</td>
<td>138,269</td>
</tr>
<tr>
<td>Wage husb. &gt; half min. wage</td>
<td>1,215</td>
<td>55,444</td>
<td>87,466</td>
</tr>
<tr>
<td>Wage wife &gt; half min. wage</td>
<td>1,723</td>
<td>53,721</td>
<td>32,021</td>
</tr>
<tr>
<td>Income husb. not from self-empl.</td>
<td>8,784</td>
<td>44,937</td>
<td>28,330</td>
</tr>
<tr>
<td>Income wife not from self-empl.</td>
<td>1,814</td>
<td>43,123</td>
<td>12,216</td>
</tr>
</tbody>
</table>

**Sample selection:** The objective of our sample selection is to apply exactly the same restrictions to the PSID, CPS, and CEX. We select married households with no missing values for gender, age, and education where: 1) the husband is between 25 and 59 years old, 2) the husband works at least 260 hours per year (a quarter part-time), 3) conditional on working, the hourly wage (annual earnings divided by annual hours) is above half of the minimum wage for both spouses, and 4) income is not from self-employment.

The marital status restriction is needed in order to be consistent with the theoretical model. Restriction 1) is imposed to avoid severe sample selection in the hours and wage data due to early retirement. Restriction 2) is imposed since one quarter of part-time employment is our definition of labor force participation. Restriction 3) is imposed to reduce implausible outliers at the bottom of the wage distribution, which is particularly important since we use the variance of log wages as a measure of dispersion (see Katz and Autor, 1999, for a discussion on the importance of trimming earnings data at the bottom). Restriction 4) is imposed since the presence of self-employment income makes it difficult to distinguish between the labor and the capital share, particularly in CPS and CEX data, and to deal with negative labor income.

Table A1 details the sample selection process in the three data sets, step by step. The final sample has 43,123 household/year observations in the PSID, 600,326 household/year observations in the CPS, and 21,556 household/year observations in the CEX.

**Top-coding:** After imposing our selection criteria, there are only 6 top-coded observations in the final PSID sample. Since we found that none of the statistics are affected by those few values, we did not make any correction for top-coded values. Roughly 2.1% of the earnings values in the final CPS sample are top-coded. Top-coding of earnings in CPS changed substantially over the sample period. We follow Katz and Autor (1999) and multiply all top-coded
Table A2: Comparison Across PSID, CPS, and CEX Samples

<table>
<thead>
<tr>
<th></th>
<th>PSID</th>
<th>CPS</th>
<th>CEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average age of men</td>
<td>39.15</td>
<td>40.94</td>
<td>41.26</td>
</tr>
<tr>
<td>Average age of women</td>
<td>37.0</td>
<td>38.62</td>
<td>39.2</td>
</tr>
<tr>
<td>Fraction of male college graduates</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Fraction of female college graduates</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>Average earnings of men (1992 $)</td>
<td>39,674</td>
<td>40,182</td>
<td>38,441</td>
</tr>
<tr>
<td>Average earnings of women (1992 $)</td>
<td>15,097</td>
<td>14,199</td>
<td>15,570</td>
</tr>
<tr>
<td>Average hours worked by men</td>
<td>2,223</td>
<td>2,252</td>
<td>2,225</td>
</tr>
<tr>
<td>Average hours worked by women</td>
<td>1,258</td>
<td>1,227</td>
<td>1,286</td>
</tr>
<tr>
<td>Average hourly wage of men (1992 $)</td>
<td>18.09</td>
<td>18.44</td>
<td>17.49</td>
</tr>
<tr>
<td>Average hourly wage of women (1992 $)</td>
<td>9.55</td>
<td>9.33</td>
<td>9.83</td>
</tr>
<tr>
<td>Average household earnings (1992 $)</td>
<td>54,772</td>
<td>54,381</td>
<td>54,011</td>
</tr>
<tr>
<td>Average food consumption (1992 $)</td>
<td>4,626</td>
<td>–</td>
<td>4,082</td>
</tr>
</tbody>
</table>

observations by a factor equal to 1.5 up to 1996 and made no correction after 1996, when top-coding observations take on the average value of all top-coded observations, by demographic group instead of the threshold value. We tried with smaller and larger factors, and our findings remain robust. In the final CEX sample there are 362 top-coded observations, i.e., around 1.7% of the total. Since the top-coding changes virtually in the same ways as in the CPS, including the change of approach after 1996, we used the Katz-Autor strategy for the CEX as well.

Comparison across data sets: Table A2 shows that–over the period where they overlap (1980-2003)–the three samples are remarkably similar in their demographic and education structure by gender. Also means of wages, earnings, and hours, by gender, are extremely similar in the three data sets. Finally, average food consumption expenditures in the PSID are very comparable to the CEX estimate.

College graduation data: The data on college graduation that we use for the calibration of the model refer to the percentage of individuals who have completed college, by gender, age group, and year from 1940 to 2006. The source is Table A.2 of the Educational Attainment section on the US Census Bureau website, [www.census.gov/population/www/socdemo](http://www.census.gov/population/www/socdemo).

A1.1 Comparison between the PSID and the CPS

Figures A1 to A4 compare the time trends in some of the key moments of the joint distribution of hours, wages, and earnings in the CPS and the PSID. The plots show deviations from the means, with means reported in the legend. These four figures demonstrate that, overall, PSID
and CPS data line up remarkably well along the vast majority of moments, in terms of both
trends and levels. The PSID moments are more volatile, due to the much smaller (by over a
factor of 15) sample size.

We find that some discrepancies in the trends of a couple of the moments involving women
arise toward the end of the sample, when the PSID data are still in “early release” format: the
female college premium (Figure A1, panel D), and the correlation between male and female log
wages (Figure A4, panel D). The trends for the moments involving men’s data are remarkably
aligned across the two data sets. The trend in household log earnings inequality (Figure A4,
panel B)—a crucial moment in our study—is somewhat flatter in the PSID. Since the trends in
male and female earnings dispersion broadly agree in the two surveys, the smaller increase in
household earnings inequality in the PSID should be attributed to the decline in the correlation
between male and female wages in the 1990s vis-à-vis the small rise of this correlation in the
CPS over the same period. The trend of the variance of household log earnings in the CEX
lies somewhere in between the PSID and the CPS. For example, over the last two decades of
available data (1984-2003), the CEX data show a rise of 0.08 log points vis-à-vis an increase of
0.12 in the CPS and 0.05 in the PSID. See Heathcote, Perri, and Violante (2010) for further
discussion on the comparability of trends in household earnings inequality across household
surveys.

A2 Identification and estimation of the wage process

A2.1 Statistical model

In the paper we posit the following statistical model of the log wage residuals for individual $i$
of age $j$ at time $t$. For all $j, t$

$$y_{i,j,t} = \eta_{i,j,t} + v_{i,j,t} + \tilde{v}_{i,j,t},$$

where $\tilde{v}_{i,j,t} \sim (0, \lambda_{\tilde{v}})$ is a transitory (i.e., uncorrelated over time) component capturing mea-

surement error in hourly wages, $v_{i,j,t} \sim (0, \lambda_v)$ is a transitory component representing genuine
individual productivity shock, and $\eta_{i,j,t}$ is the persistent component of labor productivity. In
turn, this persistent component is modelled as follows. For all $j, t > 1$

$$\eta_{i,j,t} = \rho \eta_{i,j-1,t-1} + \omega_{i,j,t},$$
where $\omega_{i,j,t} \sim (0, \lambda^{
u}_t)$. For all $t$, at age $j = 1$, $\eta_{i,1,t}$ is drawn from the time-invariant initial distribution with variance $\lambda^\eta$. We assume that $\omega_{i,j,t}$, $\tilde{v}_{i,j,t}$, $v_{i,j,t}$, and $\eta_{i,1,t}$ are orthogonal to each other, and i.i.d. across individuals in the population.

The choice of this statistical model was guided by three considerations. First, the autocovariance function for wages (across ages) shows a sharp drop between lag 0 and lag 1. This pattern suggests the presence of a purely transitory component, which likely incorporates classical measurement error in wages. Second, there are strong life cycle effects in the unconditional variance of wages: in our sample, there is almost a twofold increase in the variance between age 25 and age 59. This suggests the existence of a persistent component in individual productivity. This component is modeled as an AR(1) process. Third, the nonstationarity of the wage process is captured by indexing the distributions for productivity innovations by year rather than by cohort, following the bulk of the literature which argues that cohort effects are small compared to time effects in accounting for the rise in wage inequality in the United States (e.g., Heathcote, Storesletten, and Violante, 2005).

For all $j$, at $t = 1$ the distribution of labor productivity is assumed to be in its steady-state with variances $\{\lambda_1^\nu, \lambda_1^\omega, \lambda_2^\nu, \lambda_2^\omega, \lambda_3^\nu, \lambda_3^\omega, \lambda_4^\nu, \lambda_4^\omega, \lambda^\eta, \rho\}$. This assumption is made to maintain consistency with the model’s solution and simulations. Note that some of the variances $\{\lambda_1^\nu, \lambda_1^\omega\}$ are time-varying while others $\{\lambda_2^\nu, \lambda_2^\omega\}$ are not. We restrict the variance of measurement error $\lambda_2^\nu$ to be constant for identification purposes and, as explained in the main text, we use an external estimate to identify its size.

### A2.2 Identification: an example

We now describe the identification procedure for the case where $t = 1, 2, 4$ and $j = 1, 2, 3$. This is a useful example to illustrate our case where, after a certain date, the PSID survey becomes biennial and data for some intermediate years ($t = 3$ in the example) are missing. Let $\Upsilon$ denote the $(1 \times 10)$ parameter vector $\{\lambda_1^\nu, \lambda_2^\nu, \lambda_3^\nu, \lambda_4^\nu, \lambda_1^\omega, \lambda_2^\omega, \lambda_3^\omega, \lambda_4^\omega, \lambda^\eta, \rho\}$. The key challenge is to identify parameters at date $t = 3$.

Define the theoretical moment

$$m^j_{t,t+n} (\Upsilon) = E(y_{i,j,t} \cdot y_{i,j+n,t+n}). \quad (A1)$$

The expectation operator is defined over all individuals $i$ of age $j$ at time $t$ present both at $t$ and at $t + n$. In our simple example, we have a total of 12 such moments that we can construct
from available data.

The covariance between period $t = 1$ and $t = 2$ for the entry cohort of age $j = 1$ at $t = 1$ is

$$m_{1,2}^1 = E\left[ (\eta_{i,1.1} + v_{i,1.1}) (\eta_{i,2.2} + v_{i,2.2}) \right] = \rho \lambda^n,$$

and the same covariance between period $t = 2$ and $t = 4$ is

$$m_{2,4}^1 = E\left[ (\eta_{i,1.2} + v_{i,1.2}) (\eta_{i,3.4} + v_{i,3.4}) \right] = \rho^2 \lambda^n.$$

This pair of moments identifies $(\rho, \lambda^n)$.

At $t = 1$, the variance for the entry cohort

$$m_{1,1}^1 = E\left[ (\eta_{i,1.1} + v_{i,1.1})^2 \right] = \lambda^n + \lambda_1^\epsilon$$

identifies $\lambda_1^\epsilon$ given knowledge of $\lambda^n$.

From variance of the age group $j = 2$ at time $t = 1$,

$$m_{2,1}^2 = E\left[ (\eta_{i,2.1} + v_{i,2.1})^2 \right] = \rho^2 \lambda^n + \lambda_1^\omega + \lambda_1^\epsilon,$$

we can identify $\lambda_1^\omega$, given knowledge of the initial variance $\lambda^n$ and of $\lambda_1^\epsilon$.

At $t = 2$, the two variances for age groups $j = 1, 2$,

$$m_{2,2}^1 = E\left[ (\eta_{i,1.2} + v_{i,1.2})^2 \right] = \lambda^n + \lambda_2^\epsilon$$
$$m_{2,2}^2 = E\left[ (\eta_{i,2.2} + v_{i,2.2})^2 \right] = \rho^2 \lambda^n + \lambda_2^\omega + \lambda_2^\epsilon,$$

identify $\lambda_2^\epsilon$ and $\lambda_2^\omega$.

At $t = 4$, we can construct the three variances

$$m_{4,4}^1 = E\left[ (\eta_{i,1.4} + v_{i,1.4})^2 \right] = \lambda^n + \lambda_4^\epsilon$$
$$m_{4,4}^2 = E\left[ (\eta_{i,2.4} + v_{i,2.4})^2 \right] = \rho^2 \lambda^n + \lambda_4^\omega + \lambda_4^\epsilon$$
$$m_{4,4}^3 = E\left[ (\eta_{i,3.4} + v_{i,3.4})^2 \right] = \rho^4 \lambda^n + \rho^2 \lambda_3^\omega + \lambda_4^\omega + \lambda_4^\epsilon.$$

As usual, the variance of the entrant cohort identifies $\lambda_4^\epsilon$, given knowledge of the initial variance $\lambda^n$. Comparing the variance of new cohorts with the variance of age 2 cohorts identifies $\lambda_4^\omega$, the variance of the current persistent shock. Finally, the variance of the age $j = 3$ cohort contains the variance of the persistent shock that hit at the previous date, and this allows identification of $\lambda_3^\omega$. 8
Two remarks are in order. First, we can identify $\lambda_3^\omega$ in spite of lack of data for $t = 3$ because the $\omega$ shock hitting individuals at time $t = 3$ persists into $t = 4$, a date for which observations are available. Thus, comparing wage dispersion between a new cohort and an old cohort at $t = 4$ allows us to identify $\lambda_3^\omega$, since there are no cohort effects. Second, in general, one cannot separately identify persistent and transitory shocks in the last year of the sample. Here we can, thanks, once again, to the assumption of no cohort effects in the initial variance $\lambda^n$.

The only parameter left to identify is $\lambda_3^n$. Transitory shocks at $t = 3$ do not show up in moments at any other $t$, and thus we need to impose a restriction to complete our identification. There are several possible choices. We opt for assuming that the cross-sectional variance of wages in the population in the missing years is a weighted average of the variance in the year before and in the year after. In our specific example, if we let $\tilde{m}_{t,t}$ be the cross-sectional variance of log wages at time $t$, then we assume that $\tilde{m}_{3,3} = (\tilde{m}_{2,2} + \tilde{m}_{4,4})/2$. Given our knowledge of all the parameters $\{\rho, \lambda^n, \lambda_1^\omega, \lambda_2^\omega, \lambda_3^n\}$, one can reconstruct the cross-sectional variance component due to the cumulation of the persistent shocks up to $t = 3$. The difference between the total variance and the part due to persistent shocks identifies residually the transitory component $\lambda_3^v$.

**A2.3 Estimation**

**Parameter vector:** We have available survey data for 1967-1996, 1998, 2000, and 2002. Even though, theoretically, the variance of the persistent shocks $\lambda_i^\omega$ is identified in the missing years, in practice the fact that the lack of data occurs toward the end of the sample substantially reduces the amount of information available to estimate such parameters. Moreover, as explained, identification in the missing years hinges on the no-cohort effects assumption. Therefore, we choose to take a cautious approach and estimate $\lambda_i^\omega$ only for those years when data are available. In simulating the model, we assume that the variance of the persistent shocks for the missing years is a weighted average of the two adjacent years.

Moreover, as we have explained above, separating the variances of persistent and transitory shocks in the last year of the sample hinges also upon the, arguably restrictive, assumption of no cohort effects. Therefore, we choose not to estimate these two variances for 2002, but rather we use the 2002 survey only to improve our estimation of the structural variances up to 2000 (by constructing covariances between 2002 and the previous years). To sum up, we estimate $\rho$, $\lambda^n$, and $\{\lambda_{1967}^\omega, \ldots, \lambda_{1996}^\omega, \lambda_{1998}^\omega, \lambda_{2000}^\omega; \lambda_{1967}^v, \ldots, \lambda_{1996}^v, \lambda_{1998}^v, \lambda_{2000}^v\}$ for a total of $L = 66$ parameters.
Denote by $\mathbf{\Upsilon}$ the $(L \times 1)$ parameter vector.

**Empirical moments:** Every year $t$, we group individuals in the sample into 10-year adjacent age cells indexed by $j$, the first cell being age group 29 containing all workers between 25 and 34 years old, up until the last cell for age group 54 with individuals between 50 and 59. Our sample length and age grouping imply $T = 33$ and $J = 26$. Let $m_{i,t+n}^j (\mathbf{\Upsilon})$ be the theoretical covariance between wages in the two age group/year cells determining the triple $(j,t,n)$, exactly as in (A1). For every pair $(j,t)$, let $\bar{n} (j,t)$ be the maximum number of moments involving individuals of age $j$ at time $t$ that can be constructed from the sample (taking into account the fact that some years are missing).

The moment conditions used in the estimation are of the form

$$E (t_{i,j,t,n}) \left[ \hat{y}_{i,j,t} \cdot \hat{y}_{i,j+n,t+n} - m_{i,t+n}^j (\mathbf{\Upsilon}) \right] = 0,$$

where $t_{i,j,t,n}$ is an indicator function that equals 1 if individual $i$ has observations in both periods/age groups determined by $(j,t,n)$ and zero otherwise. The empirical counterpart of these moment conditions becomes

$$\hat{m}_{i,t+n}^j - m_{i,t+n}^j (\mathbf{\Upsilon}) = 0,$$

where $\hat{m}_{i,t+n}^j = \frac{1}{I_{j,t,n}} \sum_{i=1}^{I_{j,t,n}} \hat{y}_{i,j,t} \cdot \hat{y}_{i,j+n,t+n}$ is the empirical covariance between wages for individuals of age $j$ at time $t$ and wages of the same individuals $n$ periods later. Note that $I_{j,t,n} = \sum_{i=1}^{T} t_{i,j,t,n}$, since not all individuals contribute to each moment.

**Estimator:** The estimator we use is a minimum distance estimator that solves the following minimization problem:

$$\min_{\hat{\mathbf{\Upsilon}}} [\hat{\mathbf{m}} - \mathbf{m} (\mathbf{\Upsilon})]' \mathbf{W} [\hat{\mathbf{m}} - \mathbf{m} (\mathbf{\Upsilon})],$$

where $\hat{\mathbf{m}}$ and $\mathbf{m} (\mathbf{\Upsilon})$ are the vectors of the stacked empirical and theoretical covariances with dimension $N = \sum_{j=1}^{J} \sum_{t=1}^{T} \bar{n} (j,t)$, and $\mathbf{W}$ is a $(N \times N)$ weighting matrix. In our estimation, $N = 9,634$.

To implement the estimator, we need a choice for $\mathbf{W}$. The bulk of the literature follows Altonji and Segal (1996), who found that in common applications there is a substantial small sample bias in the estimates of $\mathbf{\Upsilon}$, hence using the identity matrix for $\mathbf{W}$ is a superior strategy to using the optimal weighting matrix characterized by Chamberlain (1984). With this choice, the solution of equation (A2) reduces to a nonlinear least square problem.
Standard errors are computed by block bootstrap, using 500 replications. Bootstrap samples are drawn at the household level with each sample containing the same number of households as the original sample. Resulting standard errors thus account for arbitrary serial dependence, heteroskedasticity, and additional estimation error induced by the use of residuals from the first stage regressions.

A3 Numerical algorithm

First we describe how we pick the sequence for the scaling variable $Z_t$. Then we review the details of the timing assumptions. Next we describe how we solve for decision rules, how we compute steady states, and calibrate the parameters set endogenously. Lastly, we describe how we handle the transition phase in which the wage structure parameters $\lambda_t$ and equilibrium prices $\mathbf{p}_t = \{p_t^{m,h}, p_t^{m,l}, p_t^{f,h}, p_t^{f,l}\}$ are time-varying. In what follows we denote initial (final) steady-state variables by the subscript $\ast$ ($\ast\ast$).

A3.1 Z sequence

The assumption that the economy is open and faces a constant pre-tax world interest rate $r$ implies a constant capital-to-aggregate effective labor ratio, since

$$ r = (\alpha Z_t K_t^{\alpha-1} H_t^{1-\alpha} - \delta) \Rightarrow \frac{K_t}{H_t} = Z_t^{\frac{1}{\alpha}} \left[ \frac{1}{\alpha} (r + \delta) \right]^{\frac{1}{\alpha-1}}. \quad (A3) $$

Substituting the expression for $K_t/H_t$ into the equilibrium expressions for prices $\mathbf{p}_t$ defined in equation (11), it is clear that prices for different types of labor are functions of the technology parameters $\{Z_t, \lambda_t^S, \lambda_t^G\}$ and of the aggregate quantities of the different types of labor supplied $H_t = \left\{ H_t^{m,h}, H_t^{m,l}, H_t^{f,h}, H_t^{f,l} \right\}$. We denote these functions by $p \left( Z_t, \lambda_t^S, \lambda_t^G, H_t \right)$.

Let $H \left( \lambda_t^S, \lambda_t^G, H_t \right)$, as defined in (1), be the function defining aggregate effective labor supply. The path for $Z_t$ is assumed such that, given the initial steady-state quantities of labor input, $H_\ast$, average individual after-tax earnings for agents of working age is equal to one at each date. This implies

$$ \frac{1}{2} \int_{S, i < j < R} d\mu_t \left( 1 - \tau^\alpha \right) \left( 1 - \alpha \right) Z_t^{\frac{\alpha}{\alpha-1}} \left[ \frac{1}{\alpha} (r + \delta) \right]^{\frac{\alpha}{\alpha-1}} H \left( \lambda_t^S, \lambda_t^G, H_\ast \right) = 1. \quad (A4) $$
A3.2 Timing

Prior to 1965 we assume the economy is in an initial steady state in which parameters $\lambda_*$ and prices $p_*$ are constant. In 1965 new information is revealed and agents revise expectations: instead of thinking that $\lambda_*$ and $p_*$ will persist forever, they now foresee the exact time-varying future paths for $\{\lambda_t\}_{t=1965}^{\infty}$ and $\{p_t\}_{t=1965}^{\infty}$.

The first and last years for which we estimate $(\lambda_t^*, \lambda_t^*)$ using our PSID sample are 1967 and 2000 (see Appendix A2). The path for $\lambda_t$ is time-varying for $1967 \leq t \leq 2000$ in such a way that the wage structure in the model evolves precisely as in the data over this period. Prices are time-varying between 1965 and 1967, even though all technology parameters in $\lambda_t$ are constant, because agents adjust their education and labor supply decisions in anticipation of future changes in the wage structure. This affects the relative supplies of different types of labor, and thus, relative prices.

We assume that by 2021, prices for the four types of labor have converged to their final steady-state values, denoted $p_{**}$. These prices are such that the model replicates the observed college premium and gender gap for 2002, the last year of our PSID sample. Adjustment to the final steady state is slow because it takes time for the educational composition of the workforce to adjust to the final steady-state values, and while this adjustment is taking place, the relative supplies of different types of labor are changing.

We need to make assumptions for the path for $\lambda_t$ during the transition period. For $t > 2000$ we assume that the wage risk parameters $(\lambda_t^w, \lambda_t^w)$ are constant and equal to the estimated values for 2000, denoted $(\lambda_{**}^w, \lambda_{**}^w)$. We assume that the path for $\lambda_t^s$ over the period $2000 < t < 2021$ is such that the relative price $p_t^{m,h}/p_t^{m,l}$ is constant at the value that replicates the observed male college premium in 2002. The path for $\lambda_t^g$ is such that the model gender premium is equal at each date to that observed in 2002. Note that these assumptions imply that both $\{\lambda_t^s, \lambda_t^g\}$ and $p_t$ are time-varying between 2000 and 2021.

Recall that some parameters are calibrated internally, as described in Section 4. In addition to all these parameter values, agents need to know the sequences for equilibrium prices $\{p_t\}$ in order to solve their problems. In practice, we proceed as follows. We first solve for initial and final steady states to set the internally calibrated time-invariant parameter values, the steady-state values for the technology parameters, and to solve for the associated steady-state prices. Given these parameters and prices, we then solve for the transition in order to fill in the sequence $\{Z_t, \lambda_t^s, \lambda_t^g\}_{t=1967}^{2020}$ and $\{p_t\}_{t=1965}^{2020}$.
A3.3 Decision rules

The household decision problems are standard finite horizon dynamic programming problems. We start in the last period of life, $J$, and work backward by age. Solving for decision rules in the retirement stage of the life cycle is relatively simple, since there is no labor market risk and the only decision for the household is how to divide income between consumption and savings. Solving for decisions in the working stage of the life cycle is more challenging computationally, because the state space is large: for each household type and for each age, we need to keep track of household wealth and of the persistent and transitory stochastic components of the wage for both the husband and the wife.

We assume that the transitory shocks and the innovations to the persistent component can each take two values, but we allow the cumulated value of the persistent component to be continuous. At each age, we approximate decision rules for consumption using piecewise tri-linear functions defined over wealth and the male and female persistent components (one function for each possible combination of age and mix of education and transitory shocks within the household). We make our grid finer at low levels of wealth, and allow the number of grid points for persistent shocks to increase with age, given that our estimates indicate a high value for the autoregressive coefficient $\rho$. We use the “endogenous grid” method for Euler equation iteration, as described by Carroll (2006). The key idea is that at each point in the state space, one considers a grid over current shocks and next period wealth, and then uses the inter-temporal first-order condition to compute implied current wealth. This can be accomplished very quickly, because it avoids having to solve a nonlinear equation (the Euler equation) numerically. The method is also well suited to dealing with borrowing constraints: setting the value for next period assets at the constraint determines the “endogenous” value for current assets below which the constraint must bind. As explained by Barillas and Fernández-Villaverde (2006), it is straightforward to extend this method to the case where labor supply is endogenous, as in our economy.

For prime-working-age households, the actual number of points on our grid for individual states is 253,920 - we solve for optimal consumption and labor supply choices at each of these. This number corresponds to 4 possible education pairs, 30 values for household wealth, 23 values each for the persistent component for the husband and the wife, and 2 values each for the transitory component. Of course, this only gives us decisions for one cohort at one particular age: we ultimately need to compute decisions for 75 ages for each of 93 different cohorts. The
total number of points at which we compute decisions is thus approximately 800 million. To simulate the economy, we simulate 20,000 households for each education composition and for each cohort, and then create cross-sectional moments by weighting appropriately by education (given enrollment rates and matching probabilities) and by cohort (given survival probabilities).

### A3.4 Steady states and internal calibration

It is useful to postpone determining \((\bar{\kappa}^m, \bar{\kappa}^f, \nu^m_{\kappa}, \nu^f_{\kappa})\) and to simply assume that there exist values for these parameters that deliver the target graduation rates by gender in the two steady states, \((q^m_a, q^f_a, q^m_{\alpha}, q^f_{\alpha})\). This way, in what follows, we can avoid solving for the education decisions.

We guess values for parameters \((Z_a, Z_{\alpha}, \lambda^G_a, \lambda^G_{\alpha}, \beta, a, \psi, b)\) and equilibrium prices \((p^m_{a}, p^m_{\alpha})\). Given the production technology and the calibration strategy, these guesses are sufficient to construct the remaining steady-state prices as follows.

First, since the selection issue for men is assumed to be minor, given guesses for \((p^m_{a}, p^m_{\alpha})\) and the observed college premia in 1967 and 2002, we immediately have \((p^{m,f}_{a}, p^{m,f}_{\alpha})\). For example, if \(\Pi_{a}\) is the ratio between the average wage of male college graduates relative to male high school graduates at the start of the sample, we set

\[
p^{m,f}_{a} = p^{m,h}_{a} / \Pi_{a}. \tag{A5}
\]

Second, given the guesses for \((\lambda^G_a, \lambda^G_{\alpha})\), from (11) we can recover steady-state prices for female labor. For example, in the first steady state

\[
\frac{p^{m,h}_{a}}{p^{f,h}_{a}} = \frac{p^{m,l}_{a}}{p^{f,l}_{a}} = 1 - \frac{\lambda^G_a}{\lambda^G_{\alpha}}. \tag{A6}
\]

In the initial steady state, the solution to the household’s problem delivers a set of decision rules, as well as associated value functions \(V_{a}\) and expected start-of-working-life values \(V_{a}^0\). Then we move to the matching stage. Given the enrollment rates \((q^m_a, q^f_a)\) and the target degree of assortative matching \(\varrho\), we can compute matching probabilities \((\pi^m_a, \pi^f_a)\) using the equation defining the correlation between education levels within the household (7) and the consistency conditions of the form (5). The same logic applies to the final steady state.

At this point, we can simulate the economy to compute cross-sectional moments. We do two simulations, one for each steady state, and compute the set of statistics that correspond to our target calibration moments and equilibrium conditions. Since technology parameters
and equilibrium prices vary across steady states, so do household decisions and cross-sectional moments. We want to calibrate the model economy to replicate certain features of the US economy (e.g., mean hours worked) on average across the sample period. We implement this by computing average empirical target statistics across the sample period, and searching for parameter values such that these are reproduced in the model when averaging across the two steady-state simulations.

To verify that the guesses for prices \( (p^m_{*h}, p^{m,h}_{**}) \) are in fact consistent with equilibrium requires knowledge of each argument of the equilibrium pricing functions, since we need to verify that \( p^m_{*h} = p(Z_*, \lambda_*^S, \lambda_*^G, H_*) \). The vector of aggregate effective hours worked by each type of labor, \( H_* \), can be computed within the simulation. The technology parameters \( Z_*^{G} \) and \( \lambda_*^{G} \) are part of the guess. However, we still need to compute the implied value for \( \lambda_*^{S} \). Since, absent selection, the observed skill premium \( \Pi_* \) is equal to the price ratio \( p^m_{*h}/p^{m,d}_{*h} \), we can compute \( \lambda_*^{S} \) using the ratio of the expressions for the marginal products of male skilled and unskilled labor:

\[
\Pi_* = \frac{p^m_{*h}}{p^{m,d}_{*h}} = \frac{\lambda_*^{S}}{1 - \lambda_*^{G}} c_* \Rightarrow \lambda_*^{S} = \frac{\Pi_*}{\Pi_* + c_*}, \tag{A7}
\]

where

\[
c_* = \left[ \frac{\lambda_*^{G} H_*^{f,h} + (1 - \lambda_*^{G}) H_*^{m,h}}{\lambda_*^{G} H_*^{f,l} + (1 - \lambda_*^{G}) H_*^{m,l}} \right]^{-\frac{1}{\sigma}}. \tag{A8}
\]

To recap, we guess a vector \( (Z_*, Z_{**}, \lambda_*^{G}, \lambda_*^{G}, \beta, \alpha, \psi, b, p^m_{*h}, p^{m,h}_{**}) \), solve the model, and check whether or not the corresponding eight target calibration moments and two equilibrium conditions for prices are satisfied. The targets (see Section 4) are (i) average no-behavioral-response after-tax earnings equal to one in each steady state, (ii) replicate gender premium in each steady state, (iii) replicate average wealth to average income ratio, (iv) replicate fraction of households with zero or negative wealth, (v) replicate average household hours, and (vi) generate realistic redistribution from the pension system (see below). If any of these conditions are not satisfied at the initial guess, we use multi-dimensional Newton-Raphson methods to update the guess. Then we resolve decision rules, and resimulate, iterating in this fashion to convergence.

One parameter (and corresponding calibration target) requires more discussion: the value for the lump-sum transfer \( b \) received by all retirees. Recall that the goal is to set \( b \) so that the dispersion of discounted lifetime earnings plus pension income in the final steady state of our economy is the same as in an alternative economy featuring the actual US Old-Age Insurance
system.

To compute US social security system benefits for a model household, we first compute average monthly earnings throughout working life (AIME). The AIME value is the input for a formula that calculates social security benefits as follows: 90% of AIME up to a first threshold (bendpoint) equal to 38% of average individual earnings, plus 32% of AIME from this bendpoint to a higher bendpoint equal to 159% of average earnings, plus 15 percent of the remaining AIME exceeding this last bendpoint. These are the actual bendpoints of the US social security system in 2007.

Once we have calculated the monthly social security benefits of husband and wife within the couple, we compute household benefits $b_{iUS}$ as the maximum between: a) the sum of the two benefits and b) 1.5 times the highest of the two benefits. This rule is called the spousal benefit rule in the US pension system. We assume pension benefits in the US system are taxed at half the labor income tax rate, which is a reasonable approximation. We repeat this procedure for every household in the artificial panel and then compute the within-cohort variance of the log of lifetime household earnings plus social security. Next, we perform a similar calculation given our alternative hypothetical pension system characterized by a lump-sum pension, $b$. The desired value for $b$ is the value that equates the dispersion in discounted lifetime income across the two systems.

The last step in the steady-state stage of the solution method is to compute the education cost distribution parameters $(\bar{\kappa}, \bar{\gamma}, \bar{\nu}_{m}, \bar{\nu}_{f})$. We do this by first using equation (4) to compute expected values of education by household type in both steady states. We then solve a simple set of four nonlinear equations of the form (3), one for each gender and for each steady state, to compute the four utility cost parameters. This procedure allows us to perfectly replicate the target graduation rates by gender in 1967 and 2002.

A3.5 Transitional dynamics

Once all parameter values are known, it remains to solve for prices from 1965 (when information about future changes in the wage structure is revealed) to 2020 (the last year of transition).

We first guess sequences \( \{p_{t}^{m,h}\}_{t=1965}^{2020} \), \( \{p_{t}^{m,l}\}_{t=1965}^{1966} \), \( \{\lambda_{t}^{G}\}_{t=1967}^{2020} \). Given these guesses, we can construct prices for each type of labor at each date as follows: (i) for $t < 1965$ prices are given by $p_{*}$, (ii) for $1965 \leq t < 1967$ prices for male labor are given by the guess \( (p_{t}^{m,h}, p_{t}^{m,l}) \), while prices for female labor can be determined given $\lambda_{t}^{G} = \lambda_{*}^{G}$ using the expression for the
gender premium (A6), (iii) for 1967 ≤ t ≤ 2020, \( p_t^{m,l} \) can be readily computed given the guess \( p_t^{m,h} \) and the empirical college premium by applying (A7), while prices for female labor are implied by the guess for \( \lambda_t^G \) and equation (A6), (iv) for \( t ≥ 2021 \), prices are given by \( p_{**} \).

Given all the prices, we solve each cohort’s problem, beginning with the cohort that enters the labor force in year \( t = 1965 - j^R = 1929 \), and ending with the cohort that enters the labor force in year 2021. We then compute cohort-specific expected values \( V_t \) for each household type.

To compute cross-sectional moments, and aggregate effective hours for each type of labor, we need the education composition of the workforce at each date. Recall that in each year we set the gender-specific means of the education cost distributions so as to exactly replicate the empirical graduation rates. Given rates \( q_t^{q} \) and the target degree of assortative matching \( \rho^* \), we compute matching probabilities \( \pi_t^{q} \) and thus the education composition for the year \( t \) cohort. Given these probabilities and the values \( V_t \), we can calculate expected education values \( M_t^{q} \).

Finally we use the equilibrium schooling condition (3) to check whether the guessed enrollment rates are correct. Enrollment rates allow us to derive the household composition for each cohort.

Once we have decision rules and household composition for all cohorts, we can simulate the economy and compute time series for the model-implied gender premium and compare this to its empirical counterpart. This is the basis for updating the sequence \( \{\lambda_t^G\} \).

To establish whether the guesses for prices are consistent with equilibrium, we need to check whether the guessed prices are equal to those implied by applying the functions \( p(Z_t, \lambda_t^S, \lambda_t^G, H_t) \). To check this we need the time series \( \{Z_t\} \) and \( \{\lambda_t^S\} \) in addition to aggregate effective hours for each type of labor, \( \{H_t\} \). We generate series for \( \{H_t\} \) by simulation, and use these series, along with the (guessed) sequences for \( \{\lambda_t^G\}, \{p_t^{m,h}\}, \) and \( \{p_t^{m,l}\} \), to compute a time series \( \{\lambda_t^S\} \) using the time \( t \) equivalent of equations (A7). We then use equation (A4) to construct a time series \( \{Z_t\} \) such that in the hypothetical counterfactual that \( H_t = H_* \) for all \( t \), average individual earnings would be time-invariant. We are then in a position to compute the model-implied equilibrium price sequences.

After comparing the guessed price sequences to the model-implied price sequences, we update our guesses. We then solve again the problem for each all cohorts, resimulate, and check again for market-clearing in all labor markets, and for the appropriate gender wage gap, iterating until convergence.

Finally, we compute how the means of the education cost distribution must evolve over time to replicate observed college completion. Given the probabilities \( \pi_t^{q} \) and the values \( V_t^{0} \), we can
calculate expected education values $M_t^d$. We then use the equilibrium schooling condition (3) to reverse-engineer the mean education costs that generate $q_t^d$ given $M_t^d$.

**A3.6 The myopic version of the model**

The numerical approach for the myopic version of the model differs slightly from the perfect foresight case described above. The basic structure is the same: we guess sequences of prices and then solve for decisions cohort by cohort. To solve for decisions of a cohort that reaches the maximum age $J$ in year $t+J$, we proceed recursively, starting from the end of the life cycle. In year $t+J$, this cohort’s consumption is a function of year $t+J$ prices. However, in year $t+J-1$, this cohort assumes year $t+J-1$ prices will prevail in both years $t+J-1$ and $t+J$. Thus, to solve for this cohort’s consumption, as a function of the state variables, at age $J-1$, we first generate an anticipated age $J$ consumption function given year $t+J-1$ prices, and then use the inter-temporal Euler equation with this anticipated age $J$ consumption function as an input to solve for the true age $J-1$, date $t+J-1$ decision rule. In this fashion, we move step by step backward through the life cycle to compute decision rules at earlier and earlier ages.

**A4 Model-data comparison over the life cycle**

Although the focus of the exercise is on changes in cross-sectional inequality over time, it is useful to check the performance of the model along the life-cycle dimension. Here we report the life-cycle dynamics in the mean and variance of household earnings and consumption for the cohort that is 25-29 years old in 1980—the initial year of the consumption sample—and we compare it to the 1980 cohort in the model. See Figure A5. The model somewhat overestimates the rise in mean household earnings, mostly after age 40, but it replicates the other life-cycle facts remarkably well.

**A5 Comparison between the economy with perfect foresight and the economy with myopic beliefs**

Figures A6 to A8 plot the key cross-sectional moments analyzed in Section 5.1 of the paper. The figures contain the time series of the various moments in the economy with perfect foresight.
and in the economy with myopic beliefs. As we emphasized in the main text, the evolution of
these moments in the two economies is remarkably similar, as evident from Figures A7 and A8.

The main reason for this close correspondence is that in both economies the college enroll-
ment rate by gender is the same, since we replicate the empirical series by design. In order
to match these series, the underlying cost distributions must differ across models, as documented
in Figure A6. In particular, in the mid-1970s, when the skill premium was temporarily low, the
myopic model calls for a substantially lower utility cost of education than what estimated in the
perfect foresight economy, since it has to reproduce the same fraction of college graduates with
much worse expectations about the returns to college education. The education cost for women
is estimated to be lower than for men also early in the period, when few women went to college,
because the large gender wage and hours gaps kept monetary returns to college education low
for women. Finally note that, in both steady states, the average cost is the same under the
two expectation models: in the steady state the future is like the present, which explains why
both lines start and end together.

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Figure A1: Comparison between CPS and PSID sample of married households.
Figure A2: Comparison between CPS and PSID sample of married households.
Figure A3: Comparison between CPS and PSID Sample of Married Households
Figure A4: Comparison between CPS and PSID sample of married households.
Figure A5: Model-data comparison. Evolution of household earnings and equivalized consumption (mean and variance of the logs) over the life cycle of the cohort that is 25-29 years old in 1980.
Figure A6: Average education cost for perfect foresight and myopic beliefs.
Figure A7: Cross-sectional hours distribution: comparison between perfect foresight and myopic beliefs.
Figure A8: Further comparison between perfect foresight and myopic beliefs.