International Diversification in Debt vs Equity

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Abstract

We solve for equilibrium portfolios in a two-country, two-good dynamic stochastic general equilibrium (DSGE) model with consumption home bias. We compare two different asset trading regimes. In the first, households in each country trade equity claims on their underlying stochastic endowments; in the second, households trade locally-denominated bonds. We derive locally accurate closed-form solutions for steady-state portfolios under each regime. The model can predict realistic equity home bias and bond diversification if the intratemporal elasticity of substitution between home and foreign goods is sufficiently low. However, for commonly used parameter values, the standard two-good model *understates* bond diversification and *overstates* equity diversification.

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1 Introduction

The modern global economy is characterized by large cross-border holdings of many kinds of financial assets (Lane and Milesi-Ferretti [2005]). However, observed international diversification is often greater in bonds than in equities. For example, at the end of 2007, foreign holdings of U.S. corporate bonds amounted to 28% of the outstanding value of those bonds, while foreign holdings of U.S. equities represented only 14% of the outstanding value of U.S. stocks. Foreign holdings of U.S. Treasury securities were even higher, at 48% of outstanding value (Federal Reserve Flow of Funds). Figure 1 shows how these shares have evolved for the U.S. from 1998 to 2007. Although all shares have gradually risen as financial market integration has accelerated, the pattern across assets is consistent: foreigners tend to hold relatively larger shares of U.S. debt than equity. Large foreign holdings of domestic debt are also prominent in the U.K. (32%, U.K. Debt Management Office), France (60%, Agence France Trésor) and other OECD countries.[1]

This paper asks a simple question: can a two-country, two-good endowment model generate realistic portfolio diversification in debt *and* equity? We show that for a sufficiently low intratemporal elasticity of substitution between home and foreign goods, the model can indeed predict home bias in equities and a reasonable degree of diversification in debt. However, the elasticity needed to generate these results is at the lower end of estimates reported in the literature. If we make the common assumption that consumers are somewhat more risk-averse than logarithmic (coefficient of constant relative risk aversion of 1.5), then the intratemporal elasticity of substitution needs to be around 0.5 to generate realistic equity home bias. Even then, the model predicts too little diversification in debt for the U.S. For intratemporal elasticities greater than 1, the model counterfactually predicts that agents will short foreign bonds. For commonly used parameter values, the standard two-good model *understates* bond diversification and *overstates* equity diversification.

We conduct our analysis using a simple two-good infinite-horizon endowment economy with two stochastic shocks, one each to home and foreign endowments. We focus on characterizing steady-state portfolios. The advantage of the model's simplicity is that we can derive locally accurate closed-form expressions for steady-state portfolios for arbitrary parameter values. We adopt the solution technique of Devereux and Sutherland [2006], who show how to solve for steady-state portfolios by analyzing second-order approximations of portfolio Euler equations and first-order approximations of the non-portfolio equations of the model.

In our two-shock model, we show that either two equities *or* two bonds is sufficient to attain complete first-order risk-sharing. Therefore, introducing both equities and bonds simultaneously would result in zero *gross* positions for bonds. We proceed instead by *comparing* two different asset trading regimes. In the first, households trade only equity claims. In the second, household trade only locally-denominated bonds. In order to analyze equity and bonds simultaneously, one would need a model with at least four shocks. In fact,

[1] Anecdotal evidence suggests that overseas holdings of *real* bonds are smaller but still non-trivial. For example, foreign and international investors purchased 8% of all U.S. Treasury Inflation-Protected Securities (TIPS) auctioned between 2000 and 2008 (U.S. Treasury Office of Debt Management).

some recent work heads in this direction, as we describe below. However, we feel it is useful to understand how a “standard” two-shock model performs for debt and equity separately, and that is the contribution of this paper.

The relatively low level of equity diversification observed in the data is often referred to as the “equity home bias puzzle”, since traditional one-good international macro models predict that countries will diversify heavily in equities in order to optimally share risk. For example, Lucas [1982] predicts that each country should hold half its financial wealth in home equity and half in foreign equity. Baxter and Jermann [1997] introduce non-diversifiable labor income risk that is positively correlated with domestic equity returns, and they show that this takes the model even further from the data: the model predicts so much diversification that domestic agents would short domestic stock. Recently, however, two-good models have had some success in resolving the equity home bias puzzle. Kollmann [2006] shows that a two-good endowment economy with no labor income risk can generate equity home bias, and Heathcote and Perri [2007] offer a two-good production economy that predicts home bias even in the presence of non-diversifiable labor income risk. Hnatkovska [2008] and Evans and Hnatkovska [2005] develop models of equity diversification with a single traded good and country-specific non-traded goods. The equity regime version of our model is substantially the same as in Kollmann [2006] and Coeurdacier [2009]. However, Kollmann [2006] and Coeurdacier [2009] derive analytical solutions only for a two-period version of the model. We show formally that the two-period equilibrium portfolios are also the steady-state portfolios of the infinite-horizon model.

There has been relatively less theoretical attention to international diversification in debt. A number of recent papers introduce debt and equity together in two-good models: see Engel and Matsumoto [2006], Pavlova and Rigobon [2003], Coeurdacier et al. [2008] and Coeurdacier and Gourinchas [2008]. However, all of these studies have more than two shocks in order to avoid portfolio indeterminacy. The new shocks (in addition to standard endowment or productivity shocks, common to all the models) range from monetary shocks (Engel and Matsumoto [2006]) to investment shocks (Coeurdacier et al. [2008]) to various kinds of demand shocks (Pavlova and Rigobon [2003]). Furthermore, most of these studies introduce bonds in order to improve the predictions of the model for *equity* portfolios, rather than to study debt portfolios per se. We adopt a different (and complementary) approach. Rather than add additional shocks, we study equities and bonds *separately* and compare the implications of the model for the two asset classes.

The paper proceeds as follows. In Section 2, we present the model. In Section 3, we present and discuss our results for equilibrium portfolios under the two asset trading regimes – equity and debt. Section 4 discusses the risk-sharing properties of the model. Section 5 concludes.

2 The Model

2.1 Households

The model economy consists of two countries, denoted home (H) and foreign (F). Each country features a "Lucas tree" that delivers a stochastic endowment of a country-specific good, Y_t^i, with $i \in \{H, F\}$. Country endowments (in logs) are assumed to follow a joint AR(1) process:

$$\log Y_t^H = \rho \log Y_{t-1}^H + \epsilon_t^H \tag{1}$$
$$\log Y_t^F = \rho \log Y_{t-1}^F + \epsilon_t^F \tag{2}$$

where $0 \leq \rho < 1$ and $\epsilon_t \equiv (\epsilon_t^H, \epsilon_t^F)$ is a vector of zero-mean iid shocks with variance-covariance matrix Σ. These endowment shocks are the only source of uncertainty in the model. We do not distinguish between "capital" and "labor" income in this model, though it would be a straightforward extension to add non-diversifiable labor income.

Each country is populated with a continuum of identical households of mass 1. Households in country i have preferences defined over a country-specific composite consumption good. We assume consumption home bias, so country i places a higher weight on i as opposed to j endowment goods, $i \neq j$. Households in country i have the following preferences:

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{(C_{t+j}^i)^{1-\gamma}}{1-\gamma} \right) \tag{3}$$

where $0 < \beta < 1$ is the subjective discount factor and $\gamma > 0$ is the (constant) coefficient of relative risk aversion. C_t^i denotes country i's consumption of its composite consumption good, which is a CES aggregate of home and foreign endowment goods:

$$C_t^i = \left[(\lambda^i)^{\frac{1}{\phi}} (C_t^{i,i})^{\frac{\phi-1}{\phi}} + (1-\lambda^i)^{\frac{1}{\phi}} (C_t^{i,j})^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad, i \neq j \tag{4}$$

where $C_t^{i,j}$ denotes country i's consumption of endowment good j; $0 < \lambda^i < 1$ is the weight that i households place on their own endowment good; and $\phi > 0$ is the elasticity of substitution between H and F endowment goods. The assumption of consumption home bias imposes $1/2 < \lambda_i \leq 1$. Moreover, we focus here on the special case of *symmetric consumption home bias*, where $\lambda^H = \lambda^F \equiv \lambda$.

Prices are perfectly flexible, and there are no nominal shocks, so the law of one price holds for H and F endowment goods. Let P_t^i denote the price of endowment good i in terms of a numeraire (to be specified shortly). The consumer price index in country i, $P_{C,t}^i$, is given as follows:

$$P_{C,t}^i = \left[\lambda^i (P_t^i)^{1-\phi} + (1-\lambda^i)(P_t^j)^{1-\phi} \right]^{\frac{1}{1-\phi}} \quad, i \neq j \tag{5}$$

We take the numeraire to be the *world price index* to preserve symmetry:

$$(P_{C,t}^{H})^{\frac{1}{2}}(P_{C,t}^{F})^{\frac{1}{2}} = 1 \tag{6}$$

It is now straightforward to define the terms of trade and the real exchange rate. The (home country's) terms of trade TOT_t are defined as the price of a home endowment good (country H's export) in terms of foreign endowment goods (country H's imports):

$$TOT_t = \frac{P_t^H}{P_t^F} \tag{7}$$

We define the real exchange rate RER_t as the price of a home consumption good in terms of foreign consumption goods:

$$RER_t = \frac{P_{C,t}^H}{P_{C,t}^F} \tag{8}$$

Note that an increase in RER_t represents a real exchange rate appreciation.

In each of the asset trading regimes described below, there are two financial assets: a home asset and a foreign asset. Let $A_{t-1}^{i,j}$ denote country i's holdings of country j's asset at the end of period $t-1$, to be carried into period t. Note that asset holdings, asset prices and asset returns are all expressed in terms of the numeraire, which is the world price index. Let R_t^j denote the real (gross) return on country j's asset. The budget constraint for a representative household in country i can be written as follows:

$$A_t^{i,H} + A_t^{i,F} = A_{t-1}^{i,H}R_t^H + A_{t-1}^{i,F}R_t^F - P_{C,t}^iC_t^i \tag{9}$$

Let $W_t^i \equiv A_t^{i,H} + A_t^{i,F}$ denote country i's financial wealth at the end of period t (in terms of the numeraire). Following Devereux and Sutherland [2006], we can rewrite (9) as follows:

$$W_t^i = W_{t-1}^iR_t^F + A_{t-1}^{i,H}\left(R_t^H - R_t^F\right) - P_{C,t}^iC_t^i \tag{10}$$

Here we are taking the foreign asset to be the "reference asset", and we are rewriting both countries' budget constraints in terms of holdings of the home asset, $A_{t-1}^{i,H}$. The behavior of asset returns will be described below. A representative household in country i maximizes (3) subject to (4) and (10), taking goods prices and asset prices as given.

2.2 Asset Trading Regimes

We consider two asset trading regimes. In the first, households trade equity claims on the home and foreign endowments. In the second, households trade locally-denominated bonds.

2.2.1 Regime 1: Equities

In the equity regime, there are two tradable equity claims. Each represents a claim on one country's endowment stream. Thus, under this regime, all income risk is explicitly diversifiable. Recall that asset prices and returns are denominated in terms of the numeraire (world price index). Let $P^i_{E,t}$ denote the price of one share of country i's equity. Returns are given as follows:

$$R^i_t = \frac{P^i_{E,t} + P^i_t Y^i_t}{P^i_{E,t-1}} \tag{11}$$

We normalize the nominal supply of each equity to 1, so the following resource constraints must hold[2]:

$$A^{H,H}_t + A^{F,H}_t = P^{EH}_t \tag{12}$$
$$A^{F,F}_t + A^{H,F}_t = P^{EF}_t \tag{13}$$

2.2.2 Regime 2: Bonds

In the bond regime, there are two locally-denominated bonds. The home bond offers a constant stream of payoffs in home goods; the foreign bond offers a constant stream of payoffs in foreign goods. Let $P^i_{B,t}$ denote the price of the bond that delivers good i. Returns are given as follows:

$$R^i_t = \frac{P^i_{B,t} + P^i_t Y}{P^i_{B,t-1}} \tag{14}$$

where Y is the steady-state value of the endowment.[3] We assume that bonds are in zero net supply, so the following resource constraints must hold:

$$A^{H,H}_t + A^{F,H}_t = 0 \tag{15}$$
$$A^{F,F}_t + A^{H,F}_t = 0 \tag{16}$$

2.2.3 Equilibrium Conditions

For either trading regime, the first-order conditions for the home household can be written as follows (assuming $\lambda^H = \lambda^F = \lambda$):

[2]Note that $A^{i,j}_t$ represents country i's holdings of country j's equity measured in value terms (in terms of world goods).

[3]The dependence of bond payoffs on Y is just a convenient normalization that makes the steady-state payoff of a bond equal to the steady-state payoff of an equity. Without loss of generality, Y can be set to 1.

$$C_t^{H,H} = \lambda \left(\frac{P_t^H}{P_{C,t}^H} \right)^{-\phi} C_t^H \tag{17}$$

$$C_t^{H,F} = (1-\lambda) \left(\frac{P_t^F}{P_{C,t}^H} \right)^{-\phi} C_t^H \tag{18}$$

$$E_t \left[M_{t+1}^H R_{t+1}^F \right] = 1 \tag{19}$$

$$E_t \left[\left(C_{t+1}^H \right)^{-\gamma} \left(P_{C,t+1}^H \right)^{-1} R_{t+1}^H \right] = E_t \left[\left(C_{t+1}^H \right)^{-\gamma} \left(P_{C,t+1}^H \right)^{-1} R_{t+1}^F \right] \tag{20}$$

$$\text{where } M_{t+1}^H = \beta \left(\frac{C_{t+1}^H}{C_t^H} \right)^{-\gamma} \left(\frac{P_{C,t+1}^H}{P_{C,t}^H} \right)^{-1} \tag{21}$$

The first-order conditions for the foreign household are:

$$C_t^{F,F} = \lambda \left(\frac{P_t^F}{P_{C,t}^F} \right)^{-\phi} C_t^F \tag{22}$$

$$C_t^{F,H} = (1-\lambda) \left(\frac{P_t^H}{P_{C,t}^F} \right)^{-\phi} C_t^F \tag{23}$$

$$E_t \left[M_{t+1}^F R_{t+1}^F \right] = 1 \tag{24}$$

$$E_t \left[\left(C_{t+1}^F \right)^{-\gamma} \left(P_{C,t+1}^F \right)^{-1} R_{t+1}^H \right] = E_t \left[\left(C_{t+1}^F \right)^{-\gamma} \left(P_{C,t+1}^F \right)^{-1} R_{t+1}^F \right] \tag{25}$$

$$\text{where } M_{t+1}^F = \beta \left(\frac{C_{t+1}^F}{C_t^F} \right)^{-\gamma} \left(\frac{P_{C,t+1}^F}{P_{C,t}^F} \right)^{-1} \tag{26}$$

Note that M_{t+1}^i is the one-period-ahead stochastic discount factor for country i expressed in terms of the numeraire.

Equilibrium consumption allocations must satisfy the market-clearing conditions for goods:

$$C_t^{H,H} + C_t^{F,H} = Y_t^H \tag{27}$$

$$C_t^{F,F} + C_t^{H,F} = Y_t^F \tag{28}$$

An *equilibrium* (for a given asset trading regime) is a sequence of values for consumption ($C_t^{i,j}$, C_t^i), financial wealth (W_t^i), goods prices (P_t^i, $P_{C,t}^i$), asset prices ($P_{E,t}^i$ or $P_{B,t}^i$), asset returns (R_t^i) and portfolio holdings ($A_t^{i,H}$) such that:

1. Households' choices for consumption and portfolio holdings are optimal, taking goods prices and asset prices as given.
2. All goods and asset markets clear.
3. Expectations are formed rationally.

3 Equilibrium Portfolios

3.1 Regime 1: Equities

In the appendix, we derive the following expression for the steady-state equity portfolio, expressed as the share of the home (foreign) equity held by home (foreign) residents.[4]

$$Sh_E = Sh_E^{H,H} = Sh_E^{F,F} = \frac{\lambda\left[\gamma - 2\lambda + 1 - 2\phi\gamma(1-\lambda)\right]}{\gamma - 1 - 4\lambda(1-\lambda)(\phi\gamma - 1)} \tag{29}$$

There is home bias in equity holdings if $Sh_E > 1/2$. To build some intuition, we consider a few special cases. We then calculate portfolio shares for different values of the structural parameters and discuss the conditions under which realistic home bias attains.

3.1.1 Special Case: No consumption home bias

If $\lambda = 1/2$, then households in both countries place equal weight on home and foreign endowment goods; i.e., there is no consumption home bias. In this case, (29) simplifies to:

$$Sh_E = \frac{1}{2} \tag{30}$$

Here all households hold perfectly balanced portfolios of half home equity and half foreign equity. When there is no consumption home bias, the model is equivalent to a one-good model, and the real exchange rate always equals unity. The only sources of uncertainty are the endowment shocks themselves, which are perfectly correlated with the equity payoffs. Thus, as in Lucas [1982], agents achieve complete risk-sharing by holding perfectly diversified portfolios.

3.1.2 Special Case: Portfolio indeterminacy with unit elasticities

If $\gamma = \phi = 1$, then preferences are logarithmic over the composite consumption good, which in turn is a Cobb-Douglas aggregate of home and foreign endowment goods. Both the intertemporal and intratemporal elasticities of substitution are 1. In this case, (29) is indeterminate. This is the famous result of Cole and Obstfeld [1991]: when elasticities of substitution are unity, endogenous variation in the terms of trade provides perfect insurance against endowment shocks, and asset trade becomes redundant. In this case, complete risk-sharing always attains, and *any* market-clearing portfolio shares could be considered an equilibrium.

[4] Since we assume that equities are in positive net supply and the model is symmetric, the expression for Sh_E is also the share of home (foreign) households' financial wealth allocated to home (foreign) equity. This reasoning will not carry over to bonds, however, because bonds are in zero net supply; steady-state wealth in the bond regime is 0.

3.1.3 General Case

To get a sense for how the equity portfolio depends on ϕ, we set $\gamma = 1.5$, so households are more risk-averse than log-investors.[5] We also set $\lambda = 0.875$, corresponding to a steady-state import share of 12.5%. This is roughly the value observed for the United States in recent years; see Coeurdacier [2009]. Figure 2 plots Sh_E for values of ϕ ranging from 0.3 to 6. Note that around $\phi = 1.43$, portfolios are indeterminate; this corresponds to the Cole/Obstfeld result discussed in the previous subsection (given that γ is now 1.5). For ϕ greater than about 1.43, there is equity foreign bias. Although this is counterfactual, it is a common prediction of single-good open economy models; this is the well known "equity home bias puzzle" or "international diversification puzzle". For ϕ less than about 1.43, there is equity home bias. However, for $1 \leq \phi < 1.43$, there is *too much* equity home bias; for these parameters, home households would hold more than 100% of the home equity by shorting foreign stocks, which is counterfactual. On the other hand, for $\phi < 1$, we do see realistic predictions for home equity bias. For example, if $\phi = 0.5$, then the share of home equities held by home households is 81%, roughly the share for the U.S. in 2005.

Table 1 tabulates Sh_E for different values of γ and ϕ, holding λ constant at 0.875 as before (the value for the U.S.). Note that to get realistic home bias, we need to assume that ϕ is small, γ is small, or both. For example, we get realistic home bias if (γ, ϕ) is around $(1.5, 0.5)$ (see previous paragraph) or around $(0.5, 2.5)$. Since most estimates of γ in the literature are greater than 1, the former case is arguably more plausible. Estimates for ϕ vary widely; 0.5 is at the low end of commonly used values (see Coeurdacier [2009] and Heathcote and Perri [2007]). There are at least two ways to think of this result. The "glass half full view" is that low substitutability between domestic and foreign goods offers a way out of the equity home bias puzzle. The "glass half empty view" is that such a low elasticity of substitution is unrealistic; a better model should be able to match the data on equity diversification with a higher elasticity. We don't take a stand on this particular point here. Rather, we simply point out that a two-country, two-good endowment model requires a fairly low intratemporal elasticity of substitution (or alternatively, agents who are less risk-averse than log-investors) to generate realistic equity home bias.

Figure (3) gives a more visual depiction of the dependence on ϕ and γ. The red plane in this figure marks the level $Sh_E = 1$, and the yellow plane marks the level $Sh_E = 0.5$. Realistic values for equity home bias correspond to points between the two planes. The figure confirms that such levels are attained only if γ is low, ϕ is low, or both.

Table (2) sets $\lambda = 0.6$. This represents a more open country than the U.S. Table (2) shows that decreasing λ tends to even out the equilibrium portfolios. Cells that exhibited strong home bias when $\lambda = 0.875$ show more moderate home bias when $\lambda = 0.6$. For example, when $(\gamma, \phi) = (1.5, 0.5)$ and $\lambda = 0.875$, we have $Sh_E = 0.81$; when λ falls to 0.6, Sh_E falls to 0.57. This result is fairly intuitive: when a country is more open to trade (values home and foreign goods more evenly), households prefer to hold more balanced portfolios of home and foreign stock.[6]

[5] Estimates of γ in the literature typically fall between 1 and 2.

[6] Note that the "evening-out effect" also occurs when there is equity foreign bias. For example, when

3.2 Regime 2: Bonds

In the appendix, we derive the following expression for the steady-state bond portfolio. Here X_B denotes home (foreign) households' holdings of the home (foreign) bond, normalized by the price of the home (foreign) bond.

$$X_B = X_B^{H,H} = X_B^{F,F} = \frac{(1-\lambda)\left[1-\gamma+2\lambda\left(\gamma\phi-1\right)\right]}{\gamma} \tag{31}$$

Recall that bonds are assumed to be in zero net supply. If no home or foreign bonds are traded in equilibrium, then $X_B = 0$. If $X_B > 0$, then home households are long the home bond and short the foreign bond. If $X_B < 0$, then home households are short the home bond and long the foreign bond. $X_B < 0$ is the more realistic scenario, as it describes a world where countries actually hold each other's bonds, rather than shorting each other's bonds. Again, we consider a few special cases before characterizing the general case.

3.2.1 Special case: No consumption home bias

If $\lambda = 1/2$, then (31) simplifies to:

$$\frac{1}{2}\left(\phi-1\right) \tag{32}$$

When there is no consumption home bias, bond holdings depend entirely on ϕ, the intratemporal elasticity of substitution. If $\phi > 1$, then home households take a long position in home bonds and a short position in foreign bonds. If $\phi < 1$, then home households take a short position in home bonds and a long position in foreign bonds. If $\phi = 1$, both households take zero positions in both bonds.[7] The intuition is that the home (foreign) bond is like a long (short) bet on the terms of trade. If $\phi = 1$, then the terms of trade move one-for-one against relative endowment shocks, and bonds offer no additional scope for improved risk-sharing. If $\phi > 1$, then after a negative home endowment shock, the terms of trade improve slightly – but not by enough to fully offset the shock. So home's relative wealth falls when its endowment falls. Therefore, the home household prefers the asset that pays more when the home endowment is low – i.e., the home bond. Conversely, if $\phi < 1$, the terms of trade move *more* than one-for-one against endowment shocks. Now home's relative wealth would *decrease* following a *positive* home endowment shock due to the large terms of trade deterioration. Therefore, the home household would prefer the asset that pays more when the home endowment is high – i.e., the foreign bond.

3.2.2 Special case: Unit intertemporal and intratemporal elasticities

If $\gamma = \phi = 1$, then (31) simplifies to:

$(\gamma, \phi) = (1.5, 4)$ and $\lambda = 0.875$, we have $Sh_e = 0.39$; when λ falls to 0.6, Sh_e rises to 0.49, corresponding to more moderate (indeed very slight) equity foreign bias.

[7] Foreign households' bond positions are symmetric. For example, if $\phi > 1$, then foreign households take a long position in foreign bonds and a short position in home bonds.

$$X_B = 0 \tag{33}$$

As discussed for equities, when $\gamma = \phi = 1$, endogenous movement in the terms of trade provide perfect insurance against endowment shocks. Therefore complete risk-sharing is achieved even without asset trade (Cole/Obstfeld indeterminacy). However, in contrast with the equity regime, the optimal steady-state bond portfolio is uniquely determined: households take zero positions in both bonds. The intuition, again, is that the home (foreign) bond is like a long (short) bet on the terms of trade. In this special case, households are already completely insured against terms of trade variation. Any additional exposure via the home or foreign bond would cause households' wealth to oscillate unnecessarily in response to endowment shocks. Therefore it is optimal for households to refrain from trading bonds.

3.2.3 General Case

Paralleling our discussion of equities, we now characterize the dependence of bond portfolios on ϕ. From (31), it is clear that the bond portfolio is linear in ϕ – in sharp contrast to the equity regime. Figure 4 plots X_B for values of ϕ ranging from 0.3 to 6, holding $\gamma = 1.5$ and $\lambda = 0.875$. The value of ϕ for which bond portfolios are zero is about 0.86. For ϕ greater than 0.86, home households are long in home bonds and short in foreign bonds. In other words, for high levels of ϕ, each country shorts the other country's bond in order to increase exposure to its domestic bond. In this case, bond investments are even less diversified than in portfolio autarky! Clearly, this is counterfactual. On the other hand, for ϕ less than 0.86, home households are short in home bonds and long in foreign bonds. In this case, the model correctly predicts positive cross-holdings of bonds.

Table 3 tabulates X_B for different values of ϕ and γ, holding $\lambda = 0.875$. Given $\gamma \geq 1$, it is clear that ϕ needs to be low in order to generate bond diversification ($X_B < 0$). Any value for ϕ greater than or equal to 1 would result in home agents shorting the foreign bond, which is counterfactual. Extending our example from the equity regime, if we take $\gamma = 1.5$ and $\phi = 0.5$, then foreign holdings of home bonds are equal to about 8% of the value of home bonds.[8] Note that this value is low for the U.S. In the data, foreign holdings of U.S. corporate bonds were 28% of outstanding value at the end of 2007, and foreign holdings of U.S. Treasuries were 48% of outstanding value. Thus, for a set of parameters that matches the U.S. on equity diversification, the model understates foreign holdings of U.S. debt.

Figure 5 gives a more visual representation of the dependence of X_B on ϕ and γ. The blue plane marks the level $X_B = 0$. Points below the plane correspond to diversification in bonds, while points above the plane correspond to "anti-diversification" (shorting of the other country's bonds). Once again, it is clear that we need either low ϕ, low γ, or both to generate bond diversification.

[8]Since bonds are in zero net supply and wealth is zero in the symmetric steady-state, home holdings of foreign bonds (and equivalently, foreign holdings of home bonds) are equal to minus home holdings of home bonds.

Table 4 sets $\lambda = 0.6$, which represents a country that is more open to trade than the U.S. Greater openness to trade (lower λ) is associated with more diversified bond portfolios *if* diversification is operative ($X_B < 0$) for the parameters (ϕ, γ) under consideration. For example, for $(\phi, \gamma) = (1.5, 0.5)$, bond diversification increases from 8% when $\lambda = 0.875$ to 21% when $\lambda = 0.6$.

4 Complete First-Order Risk-Sharing

In the appendix, we show formally that home and foreign households achieve complete risk-sharing under both equity and bond asset trading regimes, to a first order approximation. Here we briefly explain why that result obtains.

Devereux and Sutherland [2006] show that the steady-state portfolios must satisfy the following condition, which is derived from a second-order approximation of home and foreign households' first-order conditions for portfolio holdings:[9]

$$E_t \left[\widehat{CDR}_{t+1} \hat{R}^X_{t+1} \right] = 0 \tag{34}$$

where the variable $\widehat{CDR}_{t+1}$ is given by:

$$\widehat{CDR}_{t+1} \equiv \hat{C}^H_{t+1} - \hat{C}^F_{t+1} + \frac{1}{\gamma} \widehat{RER}_{t+1} \tag{35}$$

$\widehat{CDR}_t$ is the log-deviation of the ratio of H- to F- marginal utility, divided by the real exchange rate (the price of home consumption in terms of foreign consumption).[10] The model exhibits complete risk-sharing to a first-order approximation if $\widehat{CDR}_t = 0$ for all t. Since there are only two assets in this model (under either regime), the excess return $\hat{R}^X_{t+1}$ is a scalar. In the appendix, we show that the only exogenous variable that affects portfolio determination and risk-sharing is the *difference* between home and foreign endowments, $\hat{Y}_{t+1} \equiv \hat{Y}^H_{t+1} - \hat{Y}^F_{t+1}$. To a first-order approximation, then, we can express the equilibrium values for $\widehat{CDR}_{t+1}$ and $\hat{R}^X_{t+1}$ as linear combinations of (i) a vector of endogenous predetermined state variables $\hat{S}_t$, and (ii) the exogenous variable $\hat{Y}_{t+1}$.[11] That is:

$$\widehat{CDR}_{t+1} = \pi^{C,S} \hat{S}_t + \pi^{C,Y} \hat{Y}_{t+1} \tag{36}$$

$$\hat{R}^X_{t+1} = \pi^{R,S} \hat{S}_t + \pi^{R,Y} \hat{Y}_{t+1} \tag{37}$$

[9] All "hatted" variables are log-deviations from steady-state, except for $\hat{R}^X_{t+1}$, which we define to equal $\hat{R}^H_{t+1} - \hat{R}^F_{t+1}$.

[10] Devereux and Sutherland [2006] define the real exchange rate as the price of foreign consumption in terms of home consumption, so our definition of $\widehat{CDR}_{t+1}$ differs from theirs by a minus sign.

[11] By "equilibrium values", we mean the values that embed the optimal steady-state portfolios. In other words, the coefficients in (36) and (37) depend on (potentially unknown) steady-state portfolio holdings. See the appendix and Devereux and Sutherland [2006] for details.

In the appendix, we show that $\hat{S}_t$ in this model is just a single endogenous state variable that is always zero if the economy starts from the steady-state.[12] We can therefore write:

$$E_t\left[\widehat{CDR}_{t+1}\hat{R}^X_{t+1}\right] = \pi^{C,Y}\pi^{R,Y}E_t\left[\left(\hat{Y}_{t+1}\right)^2\right] \tag{38}$$

Then, for (34) to hold (i.e., for portfolios to be optimal), we must have $\pi^{R,Y} = 0$, $\pi^{C,Y} = 0$ or both. But if $\pi^{R,Y} = 0$, then the returns on the two assets are always equal regardless of realized endowment shocks. In this case, portfolio holdings are indeterminate and complete risk-sharing always obtains, as in Cole and Obstfeld [1991]. In general, $\pi^{R,Y}$ will differ from zero. Whenever it does, we have $\pi^{C,Y} = 0$. From (36), this implies that $\widehat{CDR}_t = 0$ for all t; i.e., we have complete first-order risk-sharing.

It is important to note that complete *first-order* risk-sharing does not necessarily imply complete risk-sharing in a non-approximated solution, nor does it imply that asset markets are necessarily complete. For example, under the equity regime, there is incomplete risk-sharing at higher orders of approximation whenever (i) home and foreign households place different consumption weights on home and foreign goods, and (ii) preferences are not log-separable across time and goods. However, complete first-order risk-sharing means that introducing three or more assets into a two-shock model would render one or more of the assets redundant – at least for the purposes of computing steady-state portfolios using the Devereux and Sutherland [2006] methodology. To analyze equity and bond portfolios simultaneously, one would need a model with at least four shocks. This is an area of active and future research (see Coeurdacier et al. [2008] and Coeurdacier and Gourinchas [2008]).

5 Conclusion

In this paper we have shown that a two-country, two-good DSGE endowment model can generate both equity home bias and bond diversification *if* the elasticity of substitution between home and foreign goods is sufficiently low. However, the elasticity required is at the low end of estimates reported in the literature and corresponds to a world where goods are not strong substitutes across countries. Moreover, when the model is calibrated to match equity home bias for the U.S., it underpredicts the share of U.S. debt held by foreigners. A richer model with investment and/or non-traded goods could potentially match the facts for equity and bond diversification with more realistic parameter values.

Most of the literature on international portfolio choice, including this paper, has focused on steady-state portfolios. However, there are many interesting questions about portfolio dynamics that can be addressed within this class of models. For example, how should households re-balance their portfolios in response to a foreign shock? Can such re-balancing be an important propagation mechanism for international business cycles? The dynamics become even more interesting in richer models with multiple shocks and multiple assets.

[12] This is true for both the equity and bond regimes. Starting the economy at the steady-state is a natural assumption, given that we approximate the model around the steady-state.

What kinds of shocks would lead households to substitute between debt and equity within a country, and what would lead households to substitute from home to foreign holdings within an asset class? Can such models help us understand why many emerging market countries have tilted their portfolios towards U.S. debt in recent years? We hope to address these questions in future research.

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Appendix

A Solution For Equity and Bond Portfolios

Our solution approach is based on Devereux and Sutherland [2006]. They use second-order approximations of the portfolio equations (i.e., the first-order conditions for portfolio holdings) and first-order approximations of all non-portfolio equations to jointly solve for the first-order behavior of non-portfolio variables and the zero-order (steady-state) values for portfolio holdings. All non-portfolio variables are approximated around the non-stochastic steady-state. In this section, we explain in detail how we implemented this approach to derive the closed-form solutions for steady-state equity and bond holdings given in (29) and (31).

A.1 Non-Stochastic Steady-State

In our model we approximate around the *symmetric* non-stochastic steady-state, which can be characterized as follows:

$$M^H = M^F = \beta \tag{39}$$

$$R^H = R^F = \frac{1}{\beta} \tag{40}$$

$$P^H = P^F = P_C^H = P_C^F = TOT = RER = 1 \tag{41}$$

$$C^H = C^F = Y^H = Y^F = Y \tag{42}$$

$$C^{H,H} = C^{F,F} = \lambda Y \tag{43}$$

$$C^{H,F} = C^{F,H} = (1-\lambda)Y \tag{44}$$

In addition, for the equity regime we have:

$$W^H = W^F = W = P_E^H = P_E^F = P_E = \frac{\beta Y}{1-\beta} \tag{45}$$

And for the bond regime we have:

$$W^H = W^F = 0 \tag{46}$$

$$P_B^H = P_B^F = P_B = \frac{\beta Y}{1-\beta} \tag{47}$$

A.2 Terms of trade, real exchange rate, and relative consumption expenditures

Our first step is to derive first-order approximations relating the terms of trade, the real exchange rate and relative consumption expenditures to relative endowments. The resulting equations will be re-used often to derive both the equity and bond portfolios. This step parallels Coeurdacier [2009] quite closely.

Begin by combining the intratemporal first-order conditions (17), (18), (22) and (23) with the goods market-clearing conditions (27) and (28) to show that:

$$TOT_t^{-\phi} \cdot \Omega \left[\left(\frac{P_{C,t}^F}{P_{C,t}^H} \right)^{\phi} \frac{C_t^F}{C_t^H} \right] = \frac{Y_t^H}{Y_t^F} \tag{48}$$

$$\text{where } TOT_t \equiv \frac{P_t^H}{P_t^F} \text{ and } \Omega(x) \equiv \frac{\lambda + (1-\lambda)x}{\lambda x + (1-\lambda)}$$

Now define the real exchange rate RER_t and relative (home divided by foreign) consumption expenditures PC_t as follows:

$$RER_t \equiv \frac{P_{C,t}^H}{P_{C,t}^F} \text{ and } PC_t \equiv \frac{P_{C,t}^H C_t^H}{P_{C,t}^F C_t^F}$$

With these definitions in hand, we can rewrite (48) as follows:

$$TOT_t^{-\phi} \cdot \Omega \left[\frac{RER_t^{1-\phi}}{PC_t} \right] = \frac{Y_t^H}{Y_t^F} \tag{49}$$

Log-linearize (49) around the symmetric steady-state to get:

$$-\phi \widehat{TOT}_t + (2\lambda - 1) \left[(\phi - 1)\widehat{RER}_t + \widehat{PC}_t \right] = \hat{Y}_t^H - \hat{Y}_t^F \tag{50}$$

Except where stated otherwise, "hatted" variables denote log-deviations from steady-state: $\hat{X}_t \equiv \log(X_t/X)$.[13] Next, log-linearize the expressions for the price indices (5):

$$\hat{P}_{C,t}^H = \lambda \hat{P}_t^H + (1-\lambda)\hat{P}_t^F \tag{51}$$

$$\hat{P}_{C,t}^F = \lambda \hat{P}_t^F + (1-\lambda)\hat{P}_t^H \tag{52}$$

We can now express the log-linearized real exchange rate as follows:

$$\widehat{RER}_t = \hat{P}_{C,t}^H - \hat{P}_{C,t}^F = \theta \widehat{TOT}_t \tag{53}$$

$$\text{where } \theta \equiv 2\lambda - 1$$

Combining (50) and (53) and solving for $\widehat{TOT}_t$:

[13] The exceptions are $\hat{R}_t^X$ and $\hat{W}_t^i$ (for bonds); these will be defined below.

$$\widehat{TOT}_t = \theta\psi\widehat{PC}_t - \psi\hat{Y}_t^{H,F} \tag{54}$$

$$\text{where } \psi \equiv \left[\phi - \theta^2(\phi - 1)\right]^{-1} \text{ and } \hat{Y}_t^{H,F} \equiv \hat{Y}_t^H - \hat{Y}_t^F$$

Equation (54) provides a link between TOT_t and PC_t (both endogenous) and $Y_t^{H,F}$ (exogenous). We will reuse this expression repeatedly when deriving steady-state equity and bond portfolios.

Next we log-linearize the home and foreign FOCs for consumption, (19) and (24):

$$E_t\left[-\gamma(\hat{C}_{t+1}^H - \hat{C}_t^H) - (\hat{P}_{C,t+1}^H - \hat{P}_{C,t}^H) + \hat{R}_{t+1}^H\right] = 0 \tag{55}$$

$$E_t\left[-\gamma(\hat{C}_{t+1}^F - \hat{C}_t^F) - (\hat{P}_{C,t+1}^F - \hat{P}_{C,t}^F) + \hat{R}_{t+1}^H\right] = 0 \tag{56}$$

Now subtract (55) from (56), rearrange, and use the definitions of $\widehat{RER}_t$ and $\widehat{PC}_t$:

$$E_t\left[\gamma\widehat{PC}_{t+1} - (\gamma - 1)\widehat{RER}_{t+1}\right] = \gamma\widehat{PC}_t - (\gamma - 1)\widehat{RER}_t \tag{57}$$

Substitute (53) into (57):

$$E_t\left[\gamma\widehat{PC}_{t+1} - \theta(\gamma - 1)\widehat{TOT}_{t+1}\right] = \gamma\widehat{PC}_t - \theta(\gamma - 1)\widehat{TOT}_t \tag{58}$$

And now substitute (54) into (58) and rearrange to get:

$$E_t[\widehat{PC}_{t+1}] = \widehat{PC}_t + \zeta\hat{Y}_t^{H,F} \tag{59}$$

$$\text{where } \zeta \equiv \frac{\theta\psi(\gamma - 1)(1 - \rho)}{\gamma - \theta^2\psi(\gamma - 1)} \tag{60}$$

Equation (59) provides an expression for expected one-period-ahead relative consumption expenditures. We will also make use of this equation when deriving steady-state equity and bond portfolios.

A.3 Equity Portfolios

Now consider the equity trading regime. Recall that equities are assumed to be in positive net supply, so steady-state wealth under this regime is positive. The log-linearized budget constraints (10) can be written as follows:

$$\hat{W}_{t+1}^H = \frac{1}{\beta}\left(\hat{W}_t^H + \hat{R}_{t+1}^F\right) - \left(\frac{1-\beta}{\beta}\right)\left(\hat{P}_{C,t+1}^H + \hat{C}_{t+1}^H\right) + \tilde{A}^{H,H}\hat{R}_{t+1}^X \tag{61}$$

$$\hat{W}_{t+1}^F = \frac{1}{\beta}\left(\hat{W}_t^F + \hat{R}_{t+1}^F\right) - \left(\frac{1-\beta}{\beta}\right)\left(\hat{P}_{C,t+1}^F + \hat{C}_{t+1}^F\right) + \tilde{A}^{F,H}\hat{R}_{t+1}^X \tag{62}$$

$$\text{where } \tilde{A}^{i,H} \equiv \frac{A^{i,H}}{\beta W} \text{ and } \hat{R}_{t+1}^X \equiv \hat{R}_{t+1}^H - \hat{R}_{t+1}^F$$

Note that $A^{i,H}$ denotes country i's holdings of home equity. Given a solution for $\tilde{A}^{i,H}$, we have $A^{i,H} = \beta W \tilde{A}^{i,H}$, where steady-state wealth W is given by (45). $\hat{R}^X_{t+1}$ is the (log) excess return on home equity over foreign equity.

By combining the equity market-clearing conditions (12) and (13), the definition of wealth ($W^i_t = A^{i,H}_t + A^{i,F}_t$) and the steady-state equations, one can show the following:

$$\hat{W}^F_t = \hat{P}^H_{E,t} + \hat{P}^F_{E,t} - \hat{W}^H_t \tag{63}$$

$$\tilde{A}^{F,H} = \frac{1}{\beta} - \tilde{A}^{H,H} \tag{64}$$

By log-linearizing (11), we can also derive an expression for $\hat{R}^X_{t+1}$:

$$\hat{R}^X_{t+1} = \beta(\hat{P}^H_{E,t+1} - \hat{P}^F_{E,t+1}) - (\hat{P}^H_{E,t} - \hat{P}^F_{E,t}) + (1-\beta)\widehat{TOT}_{t+1} + (1-\beta)\hat{Y}^{H,F}_{t+1} \tag{65}$$

Substitute (63), (64) and (65) into the foreign budget constraint (62) to get:

$$\begin{aligned} 2\hat{P}^F_{E,t+1} - \hat{W}^H_{t+1} = \frac{1}{\beta}\left(2\hat{P}^F_{E,t} - \hat{W}^H_t\right) + \frac{1}{\beta}\hat{R}^F_{t+1} - \left(\frac{1-\beta}{\beta}\right)\left(\hat{P}^F_{C,t+1} + \hat{C}^F_{t+1}\right) \\ + \left(\frac{1-\beta}{\beta}\right)\widehat{TOT}_{t+1} + \left(\frac{1-\beta}{\beta}\right)\hat{Y}^{H,F}_{t+1} - \tilde{A}\hat{R}^X_{t+1} \end{aligned} \tag{66}$$

$$\text{where } \tilde{A} \equiv \tilde{A}^{H,H} \equiv \frac{A^{H,H}}{\beta W}$$

Now subtract (66) from (61):

$$\begin{aligned} 2\widehat{WP}^{H,F}_{t+1} = \frac{2}{\beta}\widehat{WP}^{H,F}_t - \left(\frac{1-\beta}{\beta}\right)\left(\widehat{PC}_{t+1} + \widehat{TOT}_{t+1}\right) \\ - \left(\frac{1-\beta}{\beta}\right)\hat{Y}^{H,F}_{t+1} + 2\tilde{A}\hat{R}^X_{t+1} \end{aligned} \tag{67}$$

$$\text{where } \widehat{WP}^{H,F}_t \equiv \hat{W}^H_t - \hat{P}^F_{E,t}$$

Equation (67) is just a linear combination of the home and foreign (log-linearized) budget constraints. It is also a difference equation in $\widehat{WP}^{H,F}_t$. Following Devereux and Sutherland [2006], we note that $\hat{R}^X_{t+1}$ is a mean-zero iid random variable (to a first-order approximation), so it will not affect the eigenvalues of the log-linearized system. We therefore introduce the variable ξ_{t+1}:

$$\xi_{t+1} \equiv \tilde{A}\hat{R}^X_{t+1} \tag{68}$$

where ξ_{t+1} is also a mean-zero random variable. Solving (67) forward, applying the $E_{t+1}[\cdot]$ operator, and invoking the appropriate transversality condition, we can write:

$$\sum_{i=0}^{\infty} \beta^i E_{t+1}\left[\widehat{PC}_{t+1+i} + \widehat{TOT}_{t+1+i}\right] = \frac{2}{1-\beta}\widehat{WP}_t^{H,F} - \frac{1}{1-\beta\rho}\hat{Y}_{t+1}^{H,F} + \frac{2\beta}{1-\beta}\xi_{t+1} \tag{69}$$

where we used the fact that $E_{t+1}[\xi_{t+1+i}] = 0$ for all $i > 0$. Next, we can use equations (54) and (59) to show that:

$$E_{t+1}\left[\widehat{PC}_{t+1+i} + \widehat{TOT}_{t+1+i}\right] = (1+\theta\psi)\widehat{PC}_{t+1} + \left[(1+\theta\psi)\zeta\sum_{j=0}^{i-1}\rho^j - \psi\rho^i\right]\hat{Y}_{t+1}^{H,F} \tag{70}$$

And therefore:

$$\sum_{i=0}^{\infty} \beta^i E_{t+1}\left[\widehat{PC}_{t+1+i} + \widehat{TOT}_{t+1+i}\right] = \frac{1+\theta\psi}{1-\beta}\widehat{PC}_{t+1} + \left[\frac{\beta\zeta(1+\theta\psi) - \psi(1-\beta)}{(1-\beta\rho)(1-\beta)}\right]\hat{Y}_{t+1}^{H,F} \tag{71}$$

Now substitute (71) into (69) and solve for $\widehat{PC}_{t+1}$:

$$\widehat{PC}_{t+1} = \pi_E^{PC,WP}\widehat{WP}_t^{H,F} + \pi_E^{PC,YHF}\hat{Y}_{t+1}^{H,F} + \pi_E^{PC,\xi}\xi_{t+1} \tag{72}$$
$$\text{where } \pi_E^{PC,WP} \equiv \frac{2}{1+\theta\psi}\ ,\ \pi_E^{PC,YHF} \equiv -\left[\frac{\beta\zeta(1+\theta\psi) + (1-\psi)(1-\beta)}{(1+\theta\psi)(1-\beta\rho)}\right]\ ,$$
$$\pi_E^{PC,\xi} \equiv \frac{2\beta}{1+\theta\psi}$$

Equation (72) expresses relative consumption expenditures as a function of the endogenous state variable, $\widehat{WP}_t^{H,F}$, the exogenous relative endowment, $\hat{Y}_{t+1}^{H,F}$, and the realized excess return on the home portfolio, ξ_{t+1}. To implement the technique in Devereux and Sutherland [2006], we need to find a similar expression for the variable $\widehat{CDR}_{t+1}$:

$$\widehat{CDR}_{t+1} \equiv \hat{C}_{t+1}^H - \hat{C}_{t+1}^F + \frac{1}{\gamma}\widehat{RER}_{t+1} \tag{73}$$

To do this, note that $\widehat{CDR}_{t+1}$ is related to $\widehat{PC}_{t+1}$ as follows:

$$\begin{aligned}\widehat{CDR}_{t+1} &= \widehat{PC}_{t+1} - \left(\frac{\gamma-1}{\gamma}\right)\widehat{RER}_{t+1} \\ &= \widehat{PC}_{t+1} - \theta\left(\frac{\gamma-1}{\gamma}\right)\widehat{TOT}_{t+1} \\ &= \kappa\widehat{PC}_{t+1} + \mu\hat{Y}_{t+1}^{H,F} \end{aligned} \tag{74}$$
$$\text{where } \kappa \equiv \frac{\gamma-\theta^2\psi(\gamma-1)}{\gamma}\ ,\ \mu \equiv \frac{\theta\psi(\gamma-1)}{\gamma}$$

where the last line above follows from (54). Substituting (72) into (74) gives the desired expression:

$$\widehat{CDR}_{t+1} = \pi_E^{CDR,WP} \widehat{WP}_t^{H,F} + \pi_E^{CDR,YHF} \hat{Y}_{t+1}^{H,F} + \pi_E^{CDR,\xi} \xi_{t+1} \tag{75}$$
$$\text{where } \pi_E^{CDR,WP} \equiv \kappa \pi_E^{PC,WP} \ , \ \pi_E^{CDR,YHF} \equiv \mu + \kappa \pi_E^{PC,YHF} \ ,$$
$$\pi_E^{CDR,\xi} \equiv \kappa \pi_E^{PC,\xi}$$

The next piece we need to implement Devereux and Sutherland [2006] is an expression for $\hat{R}_{t+1}^X$ in terms of state variables. Start by substituting (54) into (65) to get:

$$\hat{R}_{t+1}^X = \beta(\hat{P}_{E,t+1}^H - \hat{P}_{E,t+1}^F) - (\hat{P}_{E,t}^H - \hat{P}_{E,t}^F) + \theta\psi(1-\beta)\widehat{PC}_{t+1} + (1-\psi)(1-\beta)\hat{Y}_{t+1}^{H,F} \tag{76}$$

Equation (76) is a difference equation in $\hat{P}_{E,t}^H - \hat{P}_{E,t}^F$. Solve it forward, apply the $E_t[\cdot]$ operator, and invoke the appropriate transversality condition to get:

$$\hat{P}_{E,t}^H - \hat{P}_{E,t}^F = \theta\psi\widehat{PC}_t + \left[\frac{\theta\psi\zeta + \rho(1-\psi)(1-\beta)}{1-\beta\rho}\right]\hat{Y}_t^{H,F} \tag{77}$$

Iterate (77) forward one period and substitute it back into (76):

$$\hat{R}_{t+1}^X = -(\hat{P}_{E,t}^H - \hat{P}_{E,t}^F) + \theta\psi\widehat{PC}_{t+1} + \left[\frac{\beta\theta\psi\zeta + (1-\psi)(1-\beta)}{1-\beta\rho}\right]\hat{Y}_{t+1}^{HF} \tag{78}$$

And finally substitute (72) into (78) to get:

$$\hat{R}_{t+1}^X = \pi_E^{RX,PHF}(\hat{P}_{E,t}^H - \hat{P}_{E,t}^F) + \pi_E^{RX,WP}\widehat{WP}_t^{H,F} + \pi_E^{RX,YHF}\hat{Y}_{t+1}^{H,F} + \pi_E^{RX,\xi}\xi_{t+1} \tag{79}$$
$$\text{where } \pi_E^{RX,PHF} \equiv -1 \ , \ \pi_E^{RX,WP} \equiv \theta\psi\pi_E^{PC,WP} \ ,$$
$$\pi_E^{RX,YHF} \equiv \theta\psi\pi_E^{PC,YHF} + \frac{\beta\theta\psi\zeta + (1-\psi)(1-\beta)}{1-\beta\rho} \ , \ \pi_E^{RX,\xi} \equiv \theta\psi\pi_E^{PC,\xi} \tag{80}$$

Equation (79) expresses the excess return $\hat{R}_{t+1}^X$ as a function of the endogenous state variables, $\hat{P}_{E,t}^H - \hat{P}_{E,t}^F$ and $\widehat{WP}_t^{H,F}$, the relative endowment, $\hat{Y}_{t+1}^{H,F}$, and the excess return on the home portfolio, ξ_{t+1}.

Devereux and Sutherland [2006] show that a *second-order* approximation of the home and foreign portfolio FOCs (20) and (25) imply:

$$E_t\left[\widehat{CDR}_{t+1}\hat{R}_{t+1}^X\right] = 0 \tag{81}$$

The steady-state portfolio $\tilde{A}$ is defined to be the one that satisfies (81). To solve for $\tilde{A}$, first set $\xi_{t+1} = \tilde{A}\hat{R}_{t+1}^X$ in (79) and re-solve for $\hat{R}_{t+1}^X$:

$$\hat{R}^X_{t+1} = \frac{\pi_E^{RX,PHF}}{1-\pi_E^{RX,\xi}\tilde{A}}\left(\hat{P}^H_{E,t}-\hat{P}^F_{E,t}\right)+\frac{\pi_E^{RX,WP}}{1-\pi_E^{RX,\xi}\tilde{A}}\widehat{WP}^{H,F}_t+\frac{\pi_E^{RX,YHF}}{1-\pi_E^{RX,\xi}\tilde{A}}\hat{Y}^{H,F}_{t+1} \tag{82}$$

Next set $\xi_{t+1} = \tilde{A}\hat{R}^X_{t+1}$ in (75):

$$\widehat{CDR}_{t+1} = \pi_E^{CDR,WP}\widehat{WP}^{H,F}_t + \pi_E^{CDR,YHF}\hat{Y}^{H,F}_{t+1} + \pi_E^{CDR,\xi}\tilde{A}\hat{R}^X_{t+1} \tag{83}$$

Substituting (82) and (83) into (81), evaluating the time-t expectations, and solving for $\tilde{A}$ gives:

$$\tilde{A} = \frac{\pi_E^{CDR,YHF}}{\pi_E^{CDR,YHF}\pi_E^{RX,\xi}-\pi_E^{CDR,\xi}\pi_E^{RX,YHF}} \tag{84}$$

The solution for steady-state home (foreign) holdings of home (foreign) equity, expressed as a fraction of steady-state wealth, can be recovered from $\tilde{A}$ as follows:

$$\frac{A}{W} = \frac{A^{H,H}}{W^H} = \frac{A^{F,F}}{W^F} = \beta\tilde{A} \tag{85}$$

Evaluating (85) and simplifying gives expression (29) in the main text.

As a double-check, we also solved for $\widehat{CDR}_{t+1}$ and $\hat{R}^X_{t+1}$ computationally using an undetermined coefficients algorithm based on Uhlig [1997]. We then recomputed the steady-state portfolio share from the numerical results using (84) and (85). The results were identical using the analytical and numerical techniques.

The expression for A/W in the text (29) also coincides with the exact analytical solution derived by Kollmann [2006] in an analogous two-period model. Our result generalizes the result of Kollmann [2006] in the following sense: we have shown that the *exact* portfolios in the two-period model correspond to the *steady-state* portfolios in an infinite-horizon model. In the infinite-horizon model, the exact solutions for portfolios also have higher-order terms which capture dynamic portfolio re-balancing. These higher-order terms are beyond the scope of this paper, but they are of interest for future work.

A.4 Bond Portfolios

We now turn to the bond trading regime. The steps to solve for steady-state bond portfolios parallel the steps for equities very closely. The main difference is that unlike equities, bonds are assumed to be in zero net supply. As a result, steady-state wealth is zero.[14] As a technicality, we can no longer characterize wealth in terms of *log* deviations from steady-state; instead, we describe the dynamics of wealth in terms of level deviations.

[14] This actually follows Devereux and Sutherland [2006] more closely, since they also approximate around a point where wealth is zero.

Begin by log-linearizing the home and foreign budget constraints (10):

$$\hat{W}^H_{t+1} = \frac{1}{\beta}\hat{W}^H_t + \hat{P}^H_{t+1} + \hat{Y}^H_{t+1} - \hat{P}^H_{C,t+1} - \hat{C}^H_{t+1} + \tilde{A}^{H,H}\hat{R}^X_{t+1} \tag{86}$$

$$\hat{W}^F_{t+1} = \frac{1}{\beta}\hat{W}^F_t + \hat{P}^F_{t+1} + \hat{Y}^F_{t+1} - \hat{P}^F_{C,t+1} - \hat{C}^F_{t+1} + \tilde{A}^{F,H}\hat{R}^X_{t+1} \tag{87}$$

$$\text{where } \tilde{A}^{i,H} \equiv \frac{A^{i,H}}{\beta Y} \text{ , } \hat{R}^X_{t+1} \equiv \hat{R}^H_{t+1} - \hat{R}^F_{t+1} \text{ ,}$$

$$\hat{W}^i_t \equiv \frac{W^i_t - W}{Y} = \frac{W^i_t}{Y}$$

Since bonds are in zero net supply, we have:

$$\tilde{A}^{F,H} = -\tilde{A}^{H,H}$$

$$\hat{W}^F_t = -\hat{W}^H_t$$

We can therefore write (87) as follows:

$$-\hat{W}^H_{t+1} = -\frac{1}{\beta}\hat{W}^H_t + \hat{P}^F_{t+1} + \hat{Y}^F_{t+1} - \hat{P}^F_{C,t+1} - \hat{C}^F_{t+1} - \tilde{A}^{H,H}\hat{R}^X_{t+1} \tag{88}$$

Now subtract (88) from (86):

$$2\hat{W}^H_{t+1} = \frac{2}{\beta}\hat{W}^H_t + \widehat{TOT}_{t+1} + \hat{Y}^{H,F}_{t+1} - \widehat{PC}_{t+1} + 2\tilde{A}\hat{R}^X_{t+1} \tag{89}$$

$$\text{where } \tilde{A} \equiv \tilde{A}^{H,H} \equiv \frac{A^{H,H}}{\beta Y}$$

Now solve (89) forward, apply the $E_{t+1}[\cdot]$ operator, invoke the appropriate transversality condition, and solve for $\widehat{PC}_{t+1}$ (using (54) and (59)) to get:

$$\widehat{PC}_{t+1} = \pi^{PC,W}_B \hat{W}^H_t + \pi^{PC,YHF}_B \hat{Y}^{H,F}_{t+1} + \pi^{PC,\xi}_B \xi_{t+1} \tag{90}$$

$$\text{where } \pi^{PC,W}_B \equiv \frac{2(1-\beta)}{\beta(1-\theta\psi)} \text{ , } \pi^{PC,YHF}_B \equiv \frac{(1-\psi)(1-\beta) - \beta\zeta(1-\theta\psi)}{(1-\theta\psi)(1-\beta\rho)} \text{ ,}$$

$$\pi^{PC,\xi}_B \equiv \frac{2(1-\beta)}{1-\theta\psi}$$

Note that equation (74), relating the variable $\widehat{CDR}_{t+1}$ to $\widehat{PC}_{t+1}$, remains valid for bonds. Therefore the solution for $\widehat{CDR}_{t+1}$ is given by:

$$\widehat{CDR}_{t+1} = \pi^{CDR,W}_B \hat{W}^H_t + \pi^{CDR,YHF}_B \hat{Y}^{H,F}_{t+1} + \pi^{CDR,\xi}_B \xi_{t+1} \tag{91}$$

$$\text{where } \pi^{CDR,W}_B \equiv \kappa\pi^{PC,W}_B \text{ , } \pi^{CDR,YHF}_B \equiv \mu + \kappa\pi^{PC,YHF}_B \text{ ,}$$

$$\pi^{CDR,\xi}_B \equiv \kappa\pi^{PC,\xi}_B$$

We now seek a similar solution for $\hat{R}^X_{t+1}$. Log-linearizing the return on bonds (14), we can write:

$$\hat{R}^X_{t+1} = \beta(\hat{P}^H_{B,t+1} - \hat{P}^F_{B,t+1}) - (\hat{P}^H_{B,t} - \hat{P}^F_{B,t}) + (1-\beta)\widehat{TOT}_{t+1} \tag{92}$$

Solve (92) forward, apply the $E_t[\cdot]$ operator, invoke the appropriate transversality condition, and use equation (59) to derive:

$$\hat{P}^H_{B,t} - \hat{P}^F_{B,t} = \theta\psi\widehat{PC}_t + \left[\frac{\theta\psi\zeta - \rho\psi(1-\beta)}{1-\beta\rho}\right]\hat{Y}^{H,F}_t \tag{93}$$

Iterate (93) forward one period, substitute it back into (92), and use (54) to get:

$$\hat{R}^X_{t+1} = -(\hat{P}^H_{B,t} - \hat{P}^F_{B,t}) + \theta\psi\widehat{PC}_{t+1} + \left[\frac{\beta\theta\psi\zeta - \psi(1-\beta)}{1-\beta\rho}\right]\hat{Y}^{H,F}_{t+1} \tag{94}$$

Substituting (90) into (94) gives the solution for $\hat{R}^X_{t+1}$:

$$\hat{R}^X_{t+1} = \pi_B^{RX,PHF}\left(\hat{P}^H_{B,t} - \hat{P}^F_{B,t}\right) + \pi_B^{RX,W}\hat{W}^H_t + \pi_B^{RX,YHF}\hat{Y}^{H,F}_{t+1} + \pi_B^{RX,\xi}\xi_{t+1} \tag{95}$$
$$\text{where } \pi_B^{RX,PHF} \equiv -1 \text{ , } \pi_B^{RX,W} \equiv \theta\psi\pi_B^{PC,W} \text{ ,}$$
$$\pi_B^{RX,YHF} \equiv \theta\psi\pi_B^{PC,YHF} + \frac{\beta\theta\psi\zeta - \psi(1-\beta)}{1-\beta\rho} \text{ , } \pi_B^{RX,\xi} \equiv \theta\psi\pi_B^{PC,\xi}$$

The solution for the $\tilde{A}$ takes the same form as the solution under the equity regime. In particular, $\tilde{A}$ must solve the bond regime analog of (84):

$$\tilde{A} = \frac{\pi_B^{CDR,YHF}}{\pi_B^{CDR,YHF}\pi_B^{RX,\xi} - \pi_B^{CDR,\xi}\pi_B^{RX,YHF}} \tag{96}$$

Actual bond holdings are related to $\tilde{A}$ as follows:

$$A = A^{H,H} = A^{F,F} = \beta Y\tilde{A} \tag{97}$$

Recall that steady-state wealth W is zero under the bond regime, so the share of wealth allocated to the home bond is not a well-defined quantity. In the main text, we report home (foreign) holdings of the home (foreign) bond, divided by the price of a home (foreign) bond:

$$X_B \equiv \frac{A}{P_B} = \frac{A}{\frac{\beta Y}{1-\beta}} = (1-\beta)\tilde{A} \tag{98}$$

B Complete First-Order Risk-Sharing

In this section we demonstrate that complete risk-sharing attains under both the equity and bond asset trading regimes, to a first-order approximation. We provide a detailed proof for the equity regime. The proof for the bond regime is analogous.

Complete risk-sharing holds in general if the ratio of marginal utilities across countries equals the real exchange rate. We limit our attention here to a first-order characterization of risk-sharing. We say that complete first-order risk-sharing attains if the log-linearized ratio of marginal utilities across countries equals the log-linearized real exchange rate. This is equivalent to showing that $\widehat{CDR}_{t+1}$ is zero. We proceed in three steps. First, we show that $\widehat{CDR}_{t+1}$ does not depend on time-$(t+1)$ shocks. Instead, it depends only on the value of a single predetermined state variable. Second, we show that this state variable is constant over time. Finally, setting the state variable to zero at time 0 completes the proof.

Start by observing that equations (72), (75) and (79) provide expressions for $\widehat{PC}_{t+1}$, $\widehat{CDR}_{t+1}$ and $\hat{R}^X_{t+1}$ in terms of the predetermined variables $\widehat{WP}^{H,F}_t$ and $\hat{P}^{H,F}_{E,t}$ $(\equiv \hat{P}^H_{E,t} - \hat{P}^F_{E,t})$; the exogenous variable $\hat{Y}^{H,F}_{t+1}$; and the excess return on the home portfolio, ξ_{t+1}. ξ_{t+1} is defined to equal $\tilde{A}\hat{R}^X_{t+1}$. Substitute this into (79) to get:

$$\hat{R}^X_{t+1} = \tilde{\pi}^{RX,PHF}_E \hat{P}^{H,F}_{E,t} + \tilde{\pi}^{RX,WP}_E \widehat{WP}^{H,F}_t + \tilde{\pi}^{RX,YHF}_E \hat{Y}^{H,F}_{t+1} \tag{99}$$
$$\text{where } \tilde{\pi}^{RX,i}_E \equiv (1 - \pi^{RX,\xi}_E \tilde{A})^{-1} \pi^{RX,i}_E \;,\; i \in \{PHF, WP, YHF\}$$

Next, substitute $\xi_{t+1} \equiv \tilde{A}\hat{R}^X_{t+1}$ into (75), and replace $\hat{R}^X_{t+1}$ using (99), to get:

$$\widehat{CDR}_{t+1} = \tilde{\pi}^{CDR,PHF}_E \hat{P}^{H,F}_{E,t} + \tilde{\pi}^{CDR,WP}_E \widehat{WP}^{H,F}_t + \tilde{\pi}^{CDR,YHF}_E \hat{Y}^{H,F}_{t+1} \tag{100}$$
$$\text{where } \tilde{\pi}^{CDR,PHF}_E \equiv \pi^{CDR,\xi}_E \tilde{A} \tilde{\pi}^{RX,PHF}_E \;,$$
$$\tilde{\pi}^{CDR,WP}_E \equiv \pi^{CDR,WP}_E + \pi^{CDR,\xi}_E \tilde{A} \tilde{\pi}^{RX,WP}_E \;,$$
$$\tilde{\pi}^{CDR,YHF}_E \equiv \pi^{CDR,YHF}_E + \pi^{CDR,\xi}_E \tilde{A} \tilde{\pi}^{RX,YHF}_E$$

Now note that we have a solution for the steady-state portfolio variable $\tilde{A}$ from (84). Substituting this into the expression for $\tilde{\pi}^{CDR,YHF}_E$, it is straightforward to show that $\tilde{\pi}^{CDR,YHF}_E = 0$. Therefore $\widehat{CDR}_{t+1}$ does not depend on time-$(t+1)$ shocks, and we have:

$$\widehat{CDR}_{t+1} = \tilde{\pi}^{CDR,PHF}_E \hat{P}^{H,F}_{E,t} + \tilde{\pi}^{CDR,WP}_E \widehat{WP}^{H,F}_t \tag{101}$$

This completes the first step in the proof. Now, let $\hat{S}_t \equiv \tilde{\pi}^{CDR,PHF}_E \hat{P}^{H,F}_{E,t} + \tilde{\pi}^{CDR,WP}_E \widehat{WP}^{H,F}_t$. We will show that $\hat{S}_{t+1} = \hat{S}_t$ for all t; i.e., $\hat{S}_t$ is constant. In what follows, it will be helpful to have an expression for $\widehat{PC}_{t+1}$ analogous to (99) and (100). This is easily obtained by substituting $\xi_{t+1} \equiv \tilde{A}\hat{R}^X_{t+1}$ into (72), and replacing $\hat{R}^X_{t+1}$ using (99):

$$\widehat{PC}_{t+1} = \tilde{\pi}_E^{PC,PHF} \hat{P}_{E,t}^{H,F} + \tilde{\pi}_E^{PC,WP} \widehat{WP}_t^{H,F} + \tilde{\pi}_E^{PC,YHF} \hat{Y}_{t+1}^{H,F} \tag{102}$$
$$\text{where } \tilde{\pi}_E^{PC,PHF} \equiv \pi_E^{PC,\xi} \tilde{A} \tilde{\pi}_E^{RX,PHF},$$
$$\tilde{\pi}_E^{PC,WP} \equiv \pi_E^{PC,WP} + \pi_E^{PC,\xi} \tilde{A} \tilde{\pi}_E^{RX,WP},$$
$$\tilde{\pi}_E^{PC,YHF} \equiv \pi_E^{PC,YHF} + \pi_E^{PC,\xi} \tilde{A} \tilde{\pi}_E^{RX,YHF}$$

Next solve (67) for $\widehat{W}_{t+1}^{H,F}$, use (54), and rearrange to get:

$$\widehat{WP}_{t+1}^{H,F} = \frac{1}{\beta} \widehat{WP}_t^{H,F} - \frac{(1+\theta\psi)(1-\beta)}{2\beta} \widehat{PC}_{t+1} - \frac{(1-\psi)(1-\beta)}{2\beta} \hat{Y}_{t+1}^{H,F} + \tilde{A} \hat{R}_{t+1}^{X} \tag{103}$$

From (77) we have:

$$\hat{P}_{E,t+1}^{H,F} = \theta\psi \widehat{PC}_{t+1} + \left[\frac{\theta\psi\zeta + \rho(1-\psi)(1-\beta)}{1-\beta\rho} \right] \hat{Y}_{t+1}^{H,F} \tag{104}$$

Combining (103) and (104), we can write:

$$\begin{aligned} \hat{S}_{t+1} &= \tilde{\pi}_E^{CDR,PHF} \hat{P}_{E,t+1}^{H,F} + \tilde{\pi}_E^{CDR,WP} \widehat{WP}_{t+1}^{H,F} \\ &= \frac{1}{\beta} \tilde{\pi}_E^{CDR,WP} \widehat{WP}_t^{H,F} + \delta_1 \widehat{PC}_{t+1} + \delta_2 \hat{Y}_{t+1}^{H,F} + \tilde{\pi}_E^{CDR,WP} \tilde{A} \hat{R}_{t+1}^{X} \end{aligned} \tag{105}$$
$$\text{where } \delta_1 \equiv \theta\psi \tilde{\pi}_E^{CDR,PHF} - \frac{(1+\theta\psi)(1-\beta)}{2\beta} \tilde{\pi}_E^{CDR,WP}$$
$$\delta_2 \equiv \left[\frac{\theta\psi\zeta + \rho(1-\psi)(1-\beta)}{1-\beta\rho} \right] \tilde{\pi}_E^{CDR,PHF} - \frac{(1-\psi)(1-\beta)}{2\beta} \tilde{\pi}_E^{CDR,WP}$$

Now substitute (102) and (99) into (105) to get:

$$\hat{S}_{t+1} = \tilde{\pi}_E^{S,PHF} \hat{P}_{E,t}^{H,F} + \tilde{\pi}_E^{S,WP} \widehat{WP}_t^{H,F} + \tilde{\pi}_E^{S,YHF} \hat{Y}_{t+1}^{H,F} \tag{106}$$
$$\text{where } \tilde{\pi}_E^{S,PHF} \equiv \delta_1 \tilde{\pi}_E^{PC,PHF} + \tilde{\pi}_E^{CDR,WP} \tilde{A} \tilde{\pi}_E^{RX,PHF}$$
$$\tilde{\pi}_E^{S,WP} \equiv \frac{1}{\beta} \tilde{\pi}_E^{CDR,WP} + \delta_1 \tilde{\pi}_E^{PC,WP} + \tilde{\pi}_E^{CDR,WP} \tilde{A} \tilde{\pi}_E^{RX,WP}$$
$$\tilde{\pi}_E^{S,YHF} \equiv \delta_1 \tilde{\pi}_E^{PC,YHF} + \delta_2 + \tilde{\pi}_E^{CDR,WP} \tilde{A} \tilde{\pi}_E^{RX,YHF}$$

It is straightforward (though tedious) to show that $\tilde{\pi}_E^{S,PHF} = \tilde{\pi}_E^{CDR,PHF}$, $\tilde{\pi}_E^{S,WP} = \tilde{\pi}_E^{CDR,WP}$ and $\tilde{\pi}_E^{S,YHF} = 0$. Therefore we have:

$$\hat{S}_{t+1} = \tilde{\pi}_E^{CDR,PHF} \hat{P}_{E,t}^{H,F} + \tilde{\pi}_E^{CDR,WP} \widehat{WP}_t^{H,F} = \hat{S}_t \tag{107}$$

So we have shown that the endogenous variable $\hat{S}_t$ is constant over time. This completes the second step of the proof. To finish the proof, assume that the economy starts in the steady-state.[15] That is, at time zero, all non-portfolio variables take on their non-stochastic steady-state values (given at the start of the appendix), and $\tilde{A}_0$ equals its endogenous steady-state value (given by (84)). Therefore $\hat{S}_0 = 0$. It follows from (107) and (101) that $\widehat{CDR}_t = 0$ for all t.

The proof of complete first-order risk-sharing in the bond regime is analogous to the proof for equities. The only difference is that the state variable now depends on $\hat{P}^{H,F}_{B,t}$ ($\equiv \hat{P}^H_{B,t} - \hat{P}^F_{B,t}$) and $\hat{W}^H_t$. To be precise, the relevant constant state variable $\hat{S}_t$ for bonds is:

$$\begin{aligned} \hat{S}_t &= \tilde{\pi}_B^{CDR,PHF} \hat{P}^{H,F}_{B,t} + \tilde{\pi}_B^{CDR,W} \hat{W}^H_t \\ \text{where } \tilde{\pi}_B^{CDR,PHF} &\equiv \pi_B^{CDR,\xi} \tilde{A} \tilde{\pi}_B^{RX,PHF} \\ \tilde{\pi}_B^{CDR,W} &\equiv \pi_B^{CDR,W} + \pi_B^{CDR,\xi} \tilde{A} \tilde{\pi}_B^{RX,W} \\ \tilde{\pi}_B^{RX,i} &\equiv (1 - \pi_B^{RX,\xi} \tilde{A})^{-1} \pi_B^{RX,i} \ , \ i \in PHF, W \end{aligned} \tag{108}$$

where $\tilde{A}$ is now given by (96).

[15] This is the natural starting point for the economy, given that we approximate around the steady-state. Note that stochastic shocks begin to hit the economy at time $t = 1$.

Table 1: Calculated values for Sh_E, the share of home equity held by home households, for different values of γ and ϕ ($\lambda = 0.875$).

γ/ϕ	0.5	0.75	1	1.5	2	4	6
0.5	1.5909	1.3276	1.1667	0.9800	0.8750	0.7000	0.6364
1	0.5000	0.5000	–	0.5000	0.5000	0.5000	0.5000
1.5	0.8077	0.9211	1.1667	-3.5000	0.0000	0.3889	0.4375
2	0.8750	0.9800	1.1667	3.5000	-0.7000	0.3182	0.4016
5	0.9486	1.0363	1.1667	1.7973	24.5000	0.1522	0.3273

Table 2: Calculated values for Sh_E, the share of home equity held by home households, for different values of γ and ϕ ($\lambda = 0.6$).

γ/ϕ	0.5	0.75	1	1.5	2	4	6
0.5	0.2727	0.0000	3.0000	0.6923	0.6000	0.5342	0.5207
1	0.5000	0.5000	–	0.5000	0.5000	0.5000	0.5000
1.5	0.5676	0.6316	3.0000	0.4286	0.4648	0.4884	0.4930
2	0.6000	0.6923	3.0000	0.3913	0.4468	0.4825	0.4895
5	0.6563	0.7941	3.0000	0.3214	0.4138	0.4719	0.4832

Table 3: Calculated values for X_B, the holdings of the home bond by home households (divided by the price of a home bond), for different values of γ and ϕ ($\lambda = 0.875$).

γ/ϕ	0.5	0.75	1	1.5	2	4	6
0.5	-0.2031	-0.1484	-0.0938	0.0156	0.1250	0.5625	1.0000
1	-0.1094	-0.0547	0.0000	0.1094	0.2188	0.6563	1.0938
1.5	-0.0781	-0.0234	0.0313	0.1406	0.2500	0.6875	1.1250
2	-0.0625	-0.0078	0.0469	0.1563	0.2656	0.7031	1.1406
5	-0.0344	0.0203	0.0750	0.1844	0.2938	0.7313	1.1688

Table 4: Calculated values for X_B, the holdings of the home bond by home households (divided by the price of a home bond), for different values of γ and ϕ ($\lambda = 0.6$).

γ/ϕ	0.5	0.75	1	1.5	2	4	6
0.5	-0.3200	-0.2000	-0.0800	0.1600	0.4000	1.3600	2.3200
1	-0.2400	-0.1200	0.0000	0.2400	0.4800	1.4400	2.4000
1.5	-0.2133	-0.0933	0.0267	0.2667	0.5067	1.4667	2.4267
2	-0.2000	-0.0800	0.0400	0.2800	0.5200	1.4800	2.4400
5	-0.1760	-0.0560	0.0640	0.3040	0.5440	1.5040	2.4640

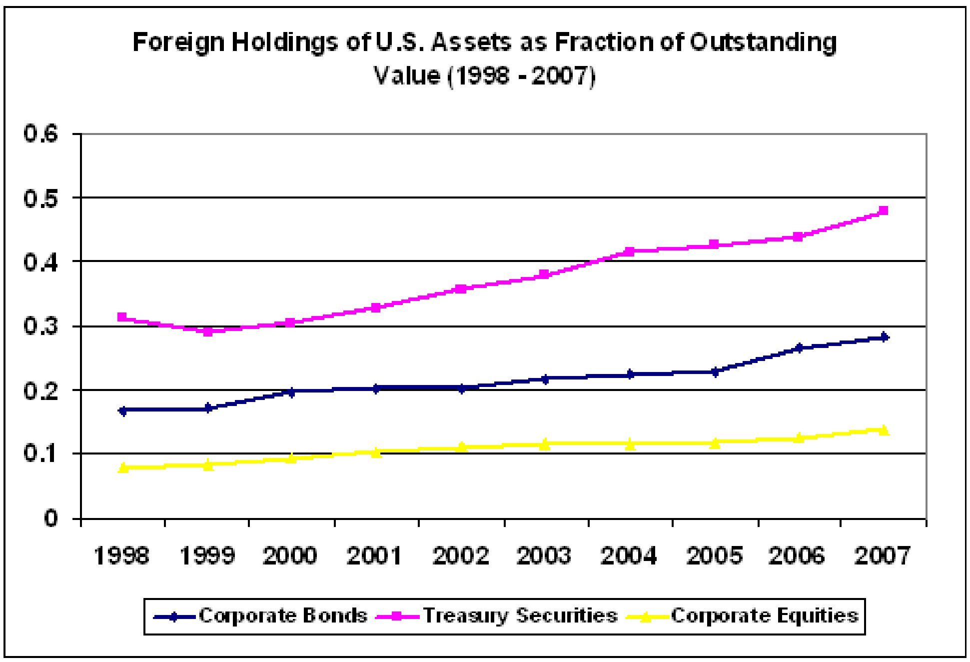

Figure 1: Foreign holdings of different U.S. financial assets, expressed as fractions of the outstanding values of those assets. Source: Federal Reserve Flow of Funds, author's calculations.

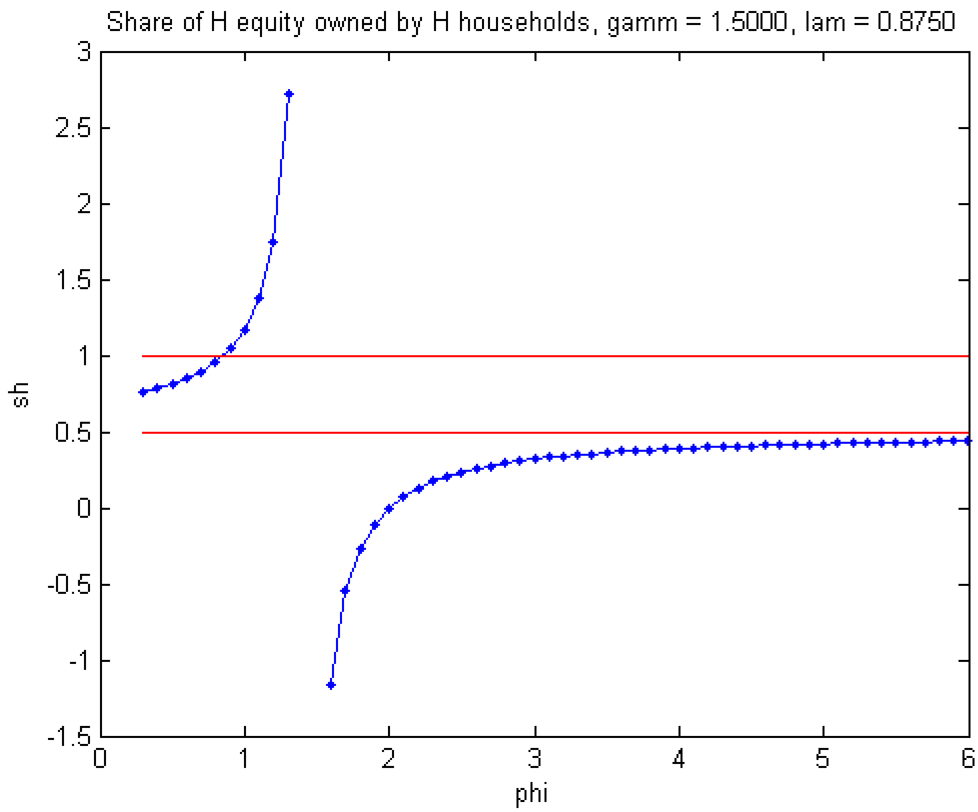

Figure 2: Share of home equity owned by home households Sh_E for different values of ϕ, the intratemporal elasticity of substitution. The coefficient of relative risk aversion, γ, is set to 1.5; and the consumption home bias parameter, λ, is set to 0.875, corresponding to a steady-state import share of 12.5%.

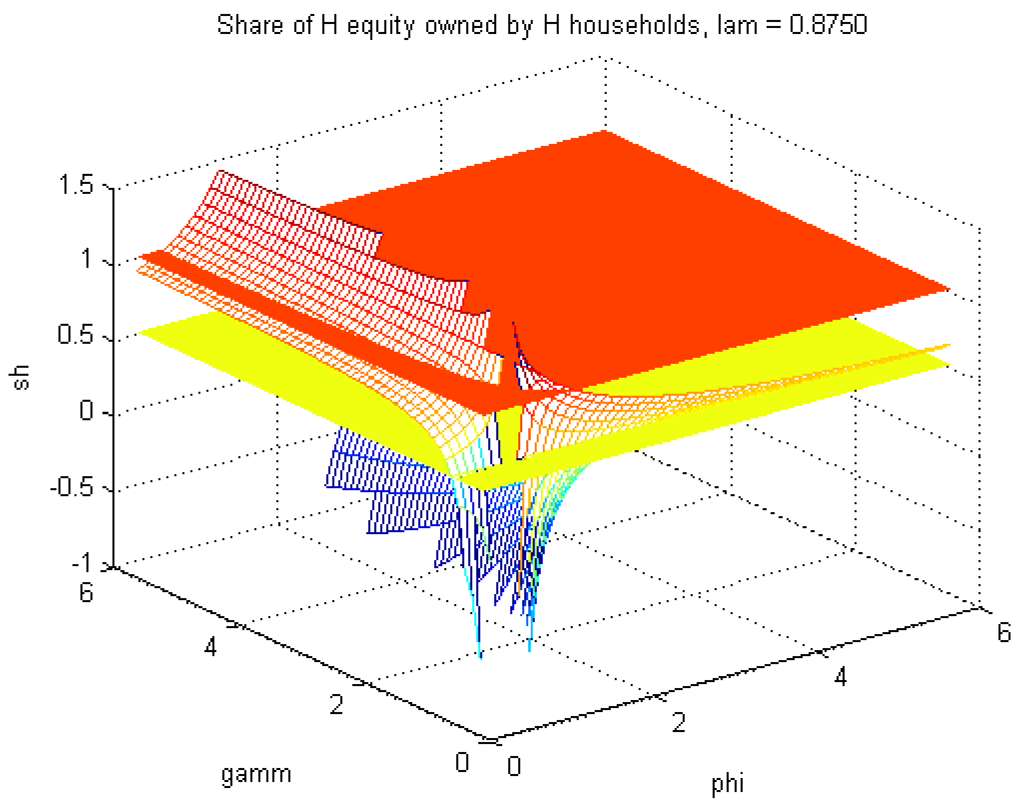

Figure 3: Share of home equity owned by home households Sh_E for different values of ϕ and γ. The consumption home bias parameter, λ, is set to 0.875, corresponding to a steady-state import share of 12.5%. The red plane marks the level $Sh_E = 1$, and the yellow plane marks the level $Sh_E = 0.5$.

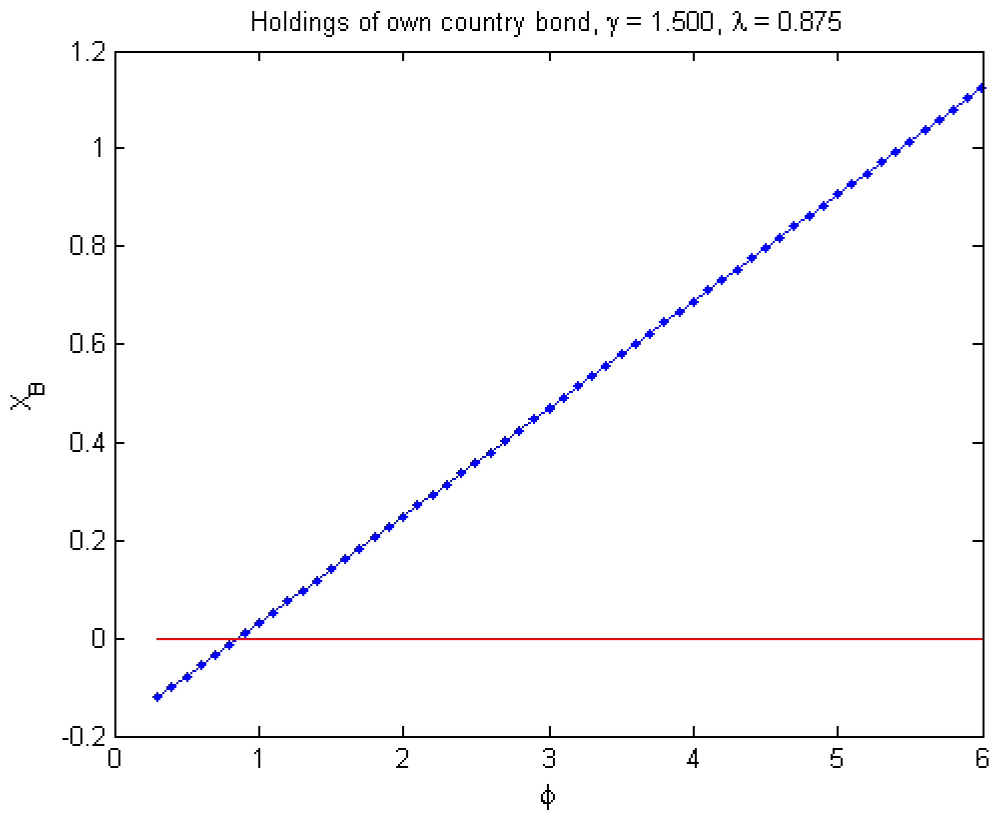

Figure 4: Home households' holdings of the home bond X_B (as a fraction of home bond price), for different values of ϕ, the intratemporal elasticity of substitution. The coefficient of relative risk aversion, γ, is set to 1.5; and the consumption home bias parameter, λ, is set to 0.875, corresponding to a steady-state import share of 12.5%.

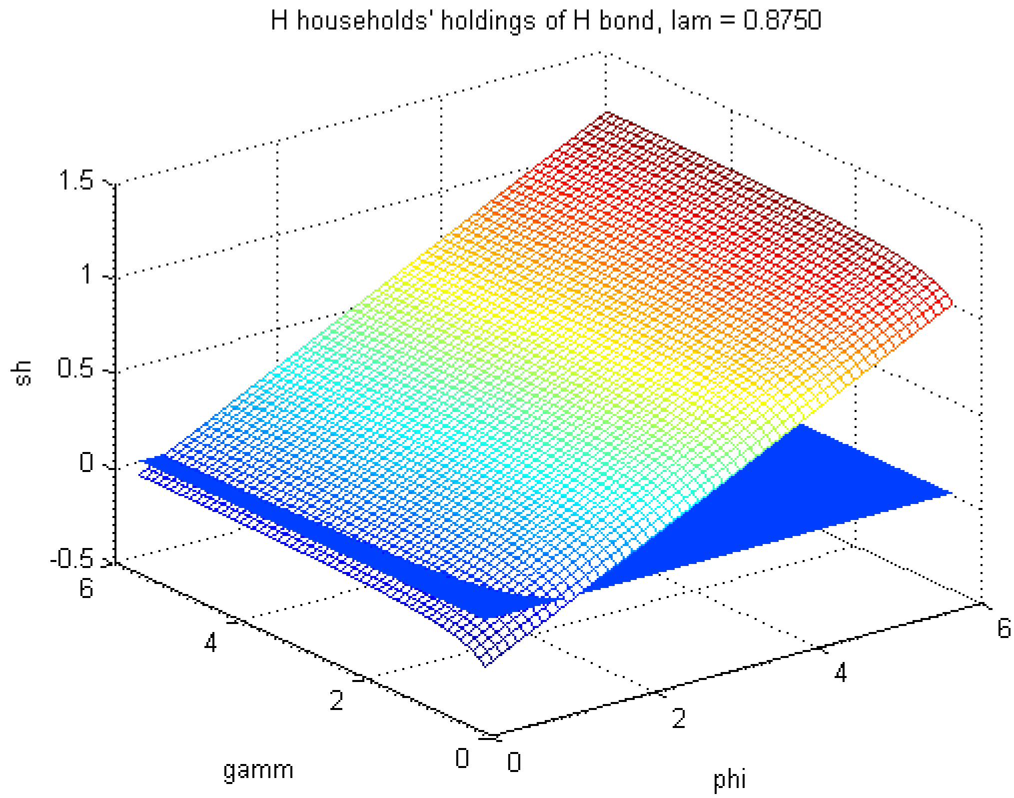

Figure 5: Home households' holdings of the home bond X_B (as a fraction of home bond price), for different values of ϕ and γ. The consumption home bias parameter, λ, is set to 0.875, corresponding to a steady-state import share of 12.5%. The blue plane marks the level $X_B = 0$.