Efficient Entry in Competing Auctions*

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Abstract

In this paper, we demonstrate the efficiency of seller entry in a model of competing auctions. We generalize the competitive search literature by simultaneously allowing for nonrival (many on one) meetings and private information. We consider both the case in which buyers learn their valuations before visiting a seller and the case in which they learn their valuations after visiting the seller. We also allow for seller heterogeneity with respect to reservation values.

1 Introduction

In this paper, we consider the efficiency of entry in a model of competitive search. By "competitive search" we mean that we analyze a large market in which buyers (or sellers) can direct their search based on the terms of trade that are posted with full commitment by their counterparts on the other side of the market. We consider in particular entry on the side of the market on which the terms of trade are advertised.

Moen (1997) and Shimer (1996) demonstrate the efficiency of entry in a competitive search market with two special features. First, meetings between buyers and sellers are assumed to be one-on-one ("rival" in the terminology used by Eeckhout and Kircher 2010). Second, there is complete information "within" any buyer-seller match, i.e., once a buyer and seller meet there is no private information. This literature shows how competition in posted wages among firms in a labor market with search frictions can

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implement the Hosios (1990) condition, i.e., the condition required for the constrained-efficient level of vacancy creation in the Diamond-Mortensen-Pissarides model (Pissarides 2000).

We generalize this literature on efficient entry in competitive search in two directions. First, we allow for many-on-one ("nonrival") meetings; i.e., a seller may interact with two or more buyers at the same time. Specifically, as is standard in the competing auctions literature, we assume urn-ball matching. Second, we allow for asymmetric information; e.g., a seller may not know how much the buyers she is interacting with value her good. With a rival meeting technology and complete information, the only relevant mechanism for selling a good is price posting. With nonrival meeting and asymmetric information, there are other mechanisms, such as auctions, to consider.

In the environment that we consider, the simplest optimal mechanism for sellers is a second-price auction. The competing auctions literature, e.g., McAfee (1993) and Peters and Severinov (1997), considers the characteristics of these auctions (in particular, reserve prices) as the market gets large. In a homogeneous-seller environment, competition drives reserve prices to the common seller reservation value. We add to this literature by considering the efficiency of seller entry.

When goods are auctioned, buyers extract an information rent from sellers. One might therefore expect that the equilibrium level of seller entry into a competitive search market would be less than the level that a social planner would choose. There is, however, a counterbalancing force, namely, that when a seller enters the market, she "steals" buyers from the sellers who were already there, and so potentially reduces the surpluses associated with these sellers. Our main result is that in a large market when the meeting technology is nonrival and when sellers are free to choose their preferred selling mechanisms, these two effects – information rent versus business stealing – exactly offset each other, leading to the socially efficient level of seller entry.

In the next section, we set up the model. We discuss two cases – first, the case in which buyers learn their valuations *ex post*, that is, only after choosing a buyer to visit, and second, the case in which buyers know their valuations *ex ante*. We show in both cases that an optimal mechanism is a second-price auction with not only a zero reserve price but also a zero participation fee. In Section 3, we present our main result that in competitive search equilibrium seller entry is constrained efficient. In this section, we also demonstrate a similar result in an environment with private information but a rival meeting technology. Section 4 extends the model to allow for seller heterogeneity with respect to reservation values and section 5 concludes.

2 The Basic Model

2.1 Environment

We consider a market with B buyers and S sellers with $B, S \to \infty$. We let $B/S = \theta$ denote "market tightness." In this market, each buyer wants to purchase one unit; each seller has one unit of the good for sale. Every seller posts and commits to a selling mechanism, and each buyer, after observing all posted mechanisms, chooses one seller from whom he will attempt to buy the good. The meeting technology is purely nonrival – the fact that one or more buyers choose to visit a particular seller does not make it more difficult for any other buyer to visit that seller. Finally, as is standard in the directed search literature, we assume that buyers cannot coordinate their visiting strategies.

We model asymmetric information in this market in a particular way. Specifically, we assume that buyer valuations for the good are distributed as $X \sim F(x)$, a continuous distribution function with corresponding density, $f(x)$. We normalize the range of X to $[0, 1]$. Buyer draws of valuations are independently and identically distributed and are private information; i.e., we are considering a model of "independent private values." We consider two cases. In the first, buyers know their valuations *ex ante*, i.e., before deciding which seller to visit. One might, for example, imagine sellers offering vacation packages. Buyers differ in their willingness to pay for this good – some are very eager to go on vacation and are willing to pay a high price if necessary; others are happy to go on vacation but only if they can do so at a low enough price. In the second case, buyers learn their valuations *ex post*, i.e., only after choosing a particular seller. This is the "inspection good" case, and in this case we treat buyer valuations as idiosyncratic match-specific random variables. Consider, for example, buyers looking to purchase in a "vertically homogeneous" segment of the housing market. Even though the houses in this market may look *ex ante* identical, a buyer, upon visiting a particular house, will have a personal idiosyncratic reaction – some buyers like houses with brightly colored wallpaper while others prefer a more subdued decor, etc.

On the seller side, we also consider two cases. First, we consider the simpler case of homogeneous sellers. In this case, we assume that all sellers have the same reservation value, which we normalize to zero. In the second case, we allow for seller heterogeneity. A fraction q of the sellers have reservation value $s > 0$ while the remaining sellers have reservation value zero. We interpret seller type as motivation – the sellers with the higher reservation value are "relaxed" about selling their good while those with reservation value zero are "desperate" (or, as is often seen in housing ads, "motivated"). Seller type is private information; q is common knowledge.

We first consider the simpler case of homogenous sellers. Our analysis of the two cases – buyers learn their valuations *ex ante* versus buyers learn their valuations *ex post* – draws heavily on Peters and Severinov (1997), who analyze "competitive matching equilibrium" in large markets for these two cases. The Peters and Severinov treatment of the *ex ante* case builds on work by McAfee (1993); their treatment of the *ex post* case extends Wolinsky (1988). This analysis is done taking market tightness as given and is a building block for the main contribution of our paper, namely, the analysis of seller entry in competitive search equilibrium.

2.2 Competitive Search Equilibrium – The Ex Post Case

Peters and Severinov (1997) consider competition in a large market in which homogeneous sellers, each with reservation value 0, post reserve prices for second-price auctions. They show that in the limit the symmetric equilibrium reserve price solves

$$\max_{r,\xi} \Pi(r,\xi) \text{ subject to } V(r,\xi) \geq \overline{V},$$

where

$$\Pi(r,\xi) = \xi \int_r^1 v(x) e^{-\xi(1-F(x))} f(x) dx \tag{1}$$

is the expected payoff that a seller can expect if she posts reserve price r, inducing a Poisson arrival rate of buyers equal to ξ, and

$$V(r,\xi) = \int_r^1 (1-F(x)) e^{-\xi(1-F(x))} dx \tag{2}$$

is the expected payoff for a buyer who chooses this seller. Here

$$v(x) = x - \frac{1-F(x)}{f(x)} \tag{3}$$

is the "virtual valuation function" and $\overline{V}$ is the expected payoff available to buyers elsewhere in the market. The constraint, $V(r,\xi) \geq \overline{V}$, expresses the idea that the "market utility property" applies in the competing auctions environment.

Peters and Severinov (1997, p.156) argue that the reserve price does not fall to zero in competitive search equilibrium. However, this turns out to be incorrect as we show in an earlier paper (Albrecht, Gautier and Vroman 2012). In the unique symmetric equilibrium, all sellers post $r = 0$ and buyers randomize their visits across sellers, i.e., $\xi = \theta$. The equilibrium allocation is thus constrained efficient – "constrained" in the sense that a social planner cannot choose which buyers to allocate to which sellers. That is, each seller who is visited by one or more buyers sells her good to the buyer with the highest valuation. In addition, given the constraint that buyers cannot coordinate their strategies, buyer randomization across sellers maximizes the expected number of transactions.

The intuition for this result is as follows. A seller chooses a reserve price to maximize her expected payoff subject to the constraint that the expected payoff for each buyer entering her auction should be no less than is available from other sellers. A seller's expected payoff is the total surplus generated by her auction minus the expected surplus of each buyer who participates in her auction. However, in a large market, expected buyer surplus is fixed by the market utility property, so it is in each seller's interest to choose the reserve price, namely $r = 0$, that maximizes total surplus.[1] In fact, a large market is not, strictly speaking, needed for this result. Levin and Smith (1994) consider a single seller offering a second-price auction with reserve price r who faces N potential buyers, each with a common outside option. Each buyer chooses to visit this seller with probability q (each buyer pays a cost if he participates in the auction, so q is endogenously determined), and the seller takes into account that reducing r increases q. Their result – endogenizing buyer entry drives r to zero, even though there is only one seller in the market – is generated by the fixed outside option, which is playing the same role as "market utility" does in the large market case.

We can augment the Peters and Severinov analysis by allowing a seller to charge a fee for participating in her auction. Now the problem for each seller is one of choosing ϕ, r and ξ to maximize

$$\phi\xi + \Pi(r,\xi) \text{ subject to } -\phi + V(r,\xi) \geq \overline{V},$$

where ϕ is the participation fee. The seller maximand reflects the fact that the seller can expect ξ participants in her auction, each paying an participation fee of ϕ; the constraint on the buyer side reflects the fact that a buyer who participates in this seller's auction has to pay the participation fee. A

[1] More generally, this intuition applies in many other competitive search models with asymmetric information, e.g., Lester, Visschers and Wolthoff (2013) and Albrecht, Gautier, and Vroman (2013). Even though the efficient mechanism may vary with the assumed environment, the market utility property gives agents who post the terms of trade the same incentive to choose an efficient mechanism.

straightforward extension of Albrecht, Gautier and Vroman (2012) shows that both ϕ and r are zero in the symmetric competitive search equilibrium and, again, that buyers randomize their visits across sellers, i.e., $\xi = \theta$.[2]

The fact that both reserve prices and participation fees are zero in a large market with a nonrival meeting technology means that not only is efficiency ensured in competitive search equilibrium but also that the division of total surplus between buyers and sellers is determined. This result is related to one derived in Gorbenko and Malenko (2011). They consider competition in "securities auctions" in which sellers auction off the right to develop projects for a combination of cash and a share of the profits. Getting buyers to pledge a profit share is a way for sellers to recapture some of the information rent associated with buyer private information. Proposition 4 in Gorbenko and Malenko (2011) shows that as the number of buyers and sellers in the market gets large, all-cash auctions are posted in the competitive search equilibrium. That is, as the market gets large, competition prevents sellers from "clawing back" any part of the information rent.

2.3 Competitive Search Equilibrium – The Ex Ante Case

Before we consider seller entry in the *ex post* case, we characterize the equilibrium in competing auctions for the case in which buyers know their valuations before choosing which seller to visit. It turns out that the competitive search equilibrium is the same as in the *ex post* case, but interestingly, the argument is quite different. Competition in selling mechanisms in the *ex ante* case was first considered in McAfee (1993). Our treatment follows Peters and Severinov (1997) and Peters (2013).

McAfee (1993) makes the following argument to show that all sellers post $r = 0$ in the *ex ante* case. Suppose buyers know their valuations before deciding which seller to visit. Suppose further that all sellers post positive reserve prices so that buyers with low valuations are shut out of the market. Now consider the seller posting the lowest reserve price, say $r' > 0$. If this seller deviates to $r = 0$, she captures the entire market between zero and r'. What is more interesting is that the deviant seller doesn't lose any buyers (in expectation) with valuations above r'. The reason is that buyers with valuations above r' don't care about competition from buyers with valuations below r'. More precisely, a buyer with valuation $x > r'$ knows that the lowest price he can possibly pay if he wins the auction is r' irrespective of whether this seller is posting $r = r'$ or $r = 0$. Buyers with valuations above

[2] Similarly, Levin and Smith (1994) show in their environment that if the seller chooses both a reserve price and an participation fee, both are set to zero.

r' continue to allocate themselves across all sellers (including the deviant) exactly as they would have absent the deviation. This argument further implies that no seller (regardless of what reserve prices are set by the other sellers) wants to set a positive reserve price. Thus, competition drives the equilibrium reserve price to zero in the *ex ante* case.

In a large market, sellers' expected payoffs are the same regardless of whether buyers learn their valuations *ex ante* or *ex post*, namely $\Pi(0, \theta)$. Expected payoffs for buyers, however are indexed by buyer type in the *ex ante* case; specifically, a buyer with valuation x can expect a payoff of

$$V(0, \theta; x) = \int_0^x e^{-\theta(1-F(y))} dy. \tag{4}$$

To see this, note first that when buyers randomize their visits, the probability that buyer x wins his auction is $e^{-\theta(1-F(x))}$. Next, conditional on winning, this buyer's expected payoff is the difference between his valuation and the expected highest draw among the other buyers participating in the same auction. Suppose n other buyers participate in the auction. All of these other buyers have valuations below x (otherwise buyer x wouldn't have won the auction), so the density of valuations across these other buyers is $f(y)/F(x)$ for $0 \leq y < x$. The expected maximum of n draws from this density is

$$E[Y_n] = \int_0^x ny \left(\frac{f(y)}{F(x)} \right) \left(\frac{F(y)}{F(x)} \right)^{n-1} dy = x - \int_0^x \left(\frac{F(y)}{F(x)} \right)^n dy,$$

so buyer x's expected payoff, conditional on winning an auction in which n other buyers participate, is $\int_0^x \left(\frac{F(y)}{F(x)} \right)^n dy$. Summing this expected payoff against the probability mass function for n, i.e., taking into account that buyers with valuations below x arrive at Poisson rate $\theta F(x)$, gives

$$\sum_{n=0}^{\infty} \frac{e^{-\theta F(x)} (\theta F(x))^n}{n!} \int_0^x \left(\frac{F(y)}{F(x)} \right)^n dy = \int_0^x e^{-\theta(F(x)-F(y))} dy.$$

Multiplying by $e^{-\theta(1-F(x))}$, i.e., the probability that buyer x wins the auction, gives equation (4). Finally, note that the average buyer payoff is

$$V(0, \theta) = \int_0^1 V(0, \theta; x) f(x) dx = \int_0^1 \int_0^x e^{-\theta(1-F(y))} dy f(x) dx = \int_0^1 (1-F(x)) e^{-\theta(1-F(x))} dx,$$

where the last equality follows by integration by parts ($u = \int_0^x e^{-\theta(1-F(y))} dy$, $v = -(1-F(x))$). That is, the expected payoff per buyer is the same as in the *ex post* case. Once we know that every seller posts $r = 0$ in the competitive search equilibrium, it is straightforward to show that competition among sellers precludes participation fees, just as in the *ex post* case.

3 Efficiency of Seller Entry – The Homogenous-Seller Case

The above characterization of competitive search equilibrium was done taking $\theta = B/S$ as given. What happens when we allow for free entry of sellers into the market? Suppose there is an entry cost (e.g., an advertising cost) of A for sellers to enter the market. The free-entry condition is

$$\Pi(0, \theta) - A = 0. \tag{5}$$

This condition holds in both cases, i.e., irrespective of whether buyers learn their valuations *ex ante* or *ex post*.

We now address the question of whether the level of seller entry is efficient in the free-entry competitive search equilibrium. The social planner problem can be posed as follows. For fixed B, choose S to maximize

$$BV(0, \theta) + S\left(\Pi(0, \theta) - A\right)$$

or equivalently, since B is fixed, choose θ to maximize

$$V(0, \theta) + \frac{\Pi(0, \theta) - A}{\theta}.$$

That is, the social planner wants to choose market tightness to maximize total market surplus, expressed on a per-buyer basis. In the *ex ante* case, this is the utilitarian social welfare function, i.e., the planner weights all buyers' expected payoffs equally. The solution to this problem is characterized by

$$V_\theta(0, \theta) + \frac{\Pi_\theta(0, \theta)\theta - (\Pi(0, \theta) - A)}{\theta^2} = 0,$$

and this holds in free-entry equilibrium since (i) $\Pi(0, \theta) - A = 0$ and (ii) $\Pi_\theta(0, \theta) + \theta V_\theta(0, \theta) = 0$. The former condition follows from free entry; the latter is the first-order condition for the seller's maximization problem as shown in Albrecht, Gautier and Vroman (2012). In short, we have shown:

Proposition 1 *In the homogeneous-seller version of the model, seller entry is constrained efficient.*

The intuition for this result is as follows. Efficient seller entry requires that the marginal entrant receive an expected payoff equal to the expected increase in market surplus generated by her entry. The marginal seller's expected payoff equals the expectation of the second highest valuation among the buyers participating in her auction (or zero if fewer than two buyers choose to participate). From the social planner's point of view, this seller's auction creates an expected increase in market surplus equal to the expected value of the highest valuation among buyers who participate in her auction (or zero if no buyers visit this seller). The difference between the expected highest and second highest valuations is an information rent, and this would seem to suggest that sellers do not have the correct incentives to enter the market. There is, however, a counterbalancing business-stealing effect. When a seller enters the market, the buyers (if any) who participate in her auction are drawn from the auctions of other sellers, so there is a decrease in surplus at the other sellers' auctions. The key to our result is that the information-rent effect and the business-stealing effect exactly offset each other in competitive search equilibrium.

To understand why these two effects are exactly offsetting, it is useful to digress to consider the payoffs for buyers and seller in the marginal seller's auction. Suppose exactly n buyers visit this seller. Then her expected payoff is $E[Y_{n-1}]$, where Y_{n-1} is the $(n-1)^{st}$ order statistic. Using standard results from order statistics,

$$E[Y_{n-1}] = E[Y_n] - n\int_0^1 F(x)^{n-1}(1 - F(x))dx,$$

where

$$E[Y_n] = \int_0^1 xnf(x)F(x)^{n-1}dx$$

is the expected value of the highest valuation drawn among these n buyers. For fixed n, the difference, $E[Y_n]-E[Y_{n-1}]$, is the information rent that goes to the winning bidder. However, since buyers are randomizing their visits across all sellers (including the marginal entrant), the number of buyers visiting any one seller is a Poisson random variable with parameter θ, and we need to take this into account in the computation of buyer and seller payoffs. We have

$$\begin{aligned}\Pi(0,\theta) &= \sum_{n=0}^{\infty}\frac{e^{-\theta}\theta^n}{n!}E[Y_{n-1}]\\ &= \sum_{n=0}^{\infty}\frac{e^{-\theta}\theta^n}{n!}\left(\int_0^1 xnf(x)F(x)^{n-1}dx - n\int_0^1 F(x)^{n-1}(1-F(x))dx\right)\\ &= \sum_{n=0}^{\infty}\frac{e^{-\theta}\theta^n}{n!}\left(\int_0^1\left(x-\frac{1-F(x)}{f(x)}\right)nf(x)F(x)^{n-1}dx\right)\\ &= \theta\int_0^1\left(x-\frac{1-F(x)}{f(x)}\right)e^{-\theta(1-F(x))}f(x)dx. \qquad (6)\end{aligned}$$

Equation (6) is the expression for expected seller payoff that is given in Peters and Severinov (1997) (and that we used above), albeit derived from a different perspective. Since second-price auctions with zero reserve prices are efficient, the total expected surplus is divided between the seller and the buyers who participate in her auction. That is,

$$\Pi(0,\theta)+\theta V(0,\theta)=\theta\int_0^1 xe^{-\theta(1-F(x))}f(x)dx = 1-\int_0^1 e^{-\theta(1-F(x))}dx, \quad (7)$$

which in turn implies that

$$V(0,\theta)=\int_0^1 (1-F(x))\,e^{-\theta(1-F(x))}dx, \quad (8)$$

which, again, is the expression for each buyer's expected payoff given in Peters and Severinov (1997). Finally, using equations (7) and (8), it is straightforward to verify that

$$\Pi_\theta(0,\theta)+\theta V_\theta(0,\theta)=0, \quad (9)$$

as we argued above.[3]

Returning now to the intuition for the efficiency of seller entry, we have shown that (i) the total surplus associated with the marginal entrant's auction is $1-\int_0^1 e^{-\theta(1-F(x))}dx$ and (ii) the difference between this total surplus and what the seller can expect to receive (the information rent) is

[3] Intuition for equation (9) can be given at the individual seller level. Consider a seller posting a second-price auction with reserve price zero. Suppose n buyers are participating in the auction. Now suppose a $(n{+}1)^{st}$ buyer also participates in the auction. Total surplus increases only if this buyer wins the auction. In this case, the buyer gets his valuation minus the value of the second highest bidder, i.e., he gets his marginal contribution to the total surplus. This means that, as equation (9) states, the sum of the payoffs to the seller and the incumbant n buyers does not change.

$\theta \int_0^1 (1 - F(x))e^{-\theta(1-F(x))}dx = \theta V(0, \theta)$. Written in this way, it is clear that the information-rent and business-stealing effects exactly cancel each other. On average, the new entrant can expect to attract θ buyers, and each of these buyers would have made an expected contribution of $V(0, \theta)$ to the surplus associated with some other seller's auction had the marginal seller not entered the market. That is, the business-stealing loss caused by the marginal seller's entry equals $\theta V(0, \theta)$.

3.1 Rival Meeting Technologies

We have shown that seller entry is efficient when buyer valuations are private information under the assumption of a nonrival meeting technology. Two earlier papers, Albrecht and Jovanovic (1986) and Guerrieri (2008), analyze competitive search models with *ex post* private information but assume a rival meeting technology.[4] A natural question is whether seller entry is also efficient when the meeting technology is rival.

To address this question, we consider the following setup. Buyers meet sellers according to a constant returns to scale meeting function $M(B, S)$ so that each buyer meets a seller with probability $\gamma(\theta)$, where $\gamma(\theta)$ satisfies $\gamma'(\theta) < 0$, $\gamma''\theta) > 0$, $\lim_{\theta \to 0} \gamma(\theta) = 1$ and $\lim_{\theta \to \infty} \gamma(\theta) = 0$. Each seller can interact with only one buyer at a time. By constant returns to scale, the probability that each seller meets a buyer is $\theta\gamma(\theta)$. When a buyer and a seller meet, the buyer draws a valuation, x, for the seller's good, where $X \sim F(x)$, with corresponding density $f(x)$, for $0 \leq x \leq 1$.

Suppose each seller posts an meeting fee, ϕ, that is, an amount the buyer has to pay before drawing his valuation, together with a purchase price, p, that is, an additional amount that is paid if the buyer purchases the good. The seller chooses ϕ, p and ξ to maximize

$$\Pi(\phi, p, \xi) = \xi\gamma(\xi) \left(\phi + (1 - F(p)) p\right)$$

subject to

$$V(\phi, p, \xi) = \gamma(\xi) \left(-\phi + \int_p^1 (1 - F(x)) \, dx\right) \geq \overline{V}.$$

It is straightforward to show (details are given in Appendix A.1) that the solution to this problem has the following properties. First, $p^* = 0$; that

[4] Faig and Jerez (2005) analyze a competitive search model with *ex post* private information and a rival meeting technology but do not consider seller entry. In the equilibrium of their model, there is pairwise efficiency, i.e., no surplus is left on the table.

is, sellers post pairwise efficient mechanisms – pairwise efficient in the sense that once a buyer and seller get together, no surplus is left on the table. The intuition for this result is the same as in the nonrival setting. By the market-utility constraint, sellers offer buyers exactly $\overline{V}$. Then, given the fixed payoff to the buyer, it is in each seller's interest to choose a mechanism that maximizes total surplus. Second, each seller's implicit choice of ξ satisfies $\Pi_\xi(\phi^*, 0, \xi^*) + \xi^* V_\xi(\phi^*, 0, \xi^*) = 0$. Since all sellers post the same mechanism, $\xi^* = \theta$, and we have $\Pi_\theta(\phi^*, 0, \theta) + \theta V_\theta(\phi^*, 0, \theta) = 0$, which, as in the nonrival case, together with the free entry condition, gives the efficient level of seller entry. Once sellers post $p^* = 0$, private information is irrelevant, and the question of efficient seller entry is the same as in models like Moen (1997). The condition, $\Pi_\theta(\phi^*, 0, \theta) + \theta V_\theta(\phi^*, 0, \theta) = 0$, expresses the idea that at the social planner optimum, the "congestion" and "thick market" externalities that come from search frictions are exactly offsetting. Finally, competitive search together with the free-entry condition determines the equilibrium meeting fee.

As noted above, related results are given in Albrecht and Jovanovic (1986) and Guerrieri (2008). These papers are both set in the labor market so that the terms of trade are posted on the buyer (vacancy) side of the market. Then the question is whether the level of vacancy creation is efficient. Albrecht and Jovanovic (1986) show that vacancies post efficient mechanisms and that the level of vacancy creation is efficient but only under the assumption that $M(U, V) = \min[U, V]$. With this meeting function, efficiency of vacancy creation reduces to ensuring that there are equal numbers of unemployed and vacancies in the market, and this condition is straightforward in their competitive search equilibrium. Guerrieri (2008) uses a general rival meeting function, but she assumes that vacancies cannot post meeting fees. In her competitive search equilibrium, the posted wage is too low, so the posted mechanism is pairwise inefficient since some matches that would yield a positive surplus are not formed. Conditional on the posted mechanism, however, the level of vacancy creation is efficient.[5] This setting leads to a counterintuitive result (Proposition 3 in Guerrieri 2008). For some parameter configurations, the social planner can increase welfare by "money burning," i.e., the planner can make workers better of by reducing the value of their outside option. This occurs because the reduction in the outside option makes workers willing to accept lower wages. This leads to

[5] More specifically, vacancy creation is efficient in the static version of her model. In the dynamic version of her model, she shows that vacancy creation is inefficient in the adjustment to the steady state.

an increase of vacancies in the market and this can make workers better off because even though their outside option has been reduced and they earn a lower wage when employed, they have a sufficiently higher probability of employment so that there is an overall increase in their expected utility. This could not occur in Albrecht and Jovanovic (1986) or in our setup because the equilibrium is constrained efficient and there is no scope to improve welfare.

4 Heterogeneous Sellers

4.1 The Ex Post Case

Suppose a fraction q of the sellers in the market are "relaxed" (Type H) with reservation value $s \in (0,1)$, while a fraction $1-q$ are "desperate" (Type L) with reservation value zero. Competition forces every Type-H seller to post a second-price auction with reserve price s. To see this, note that a Type-H seller chooses r and ξ to maximize

$$\Pi(r,\xi;s) = s + \xi \int_r^1 (v(x)-s))\, e^{-\xi(1-F(x))} f(x)dx \;\text{ s. t. }\; V(r,\xi) = \overline{V},$$

where $v(x)$ is the virtual valuation function defined in equation (3). The maximand reflects the fact that the seller loses her reservation value only if the good is sold, and the notation reflects the fact that the seller's expected payoff varies with her reservation value. An extension of the argument given in Albrecht, Gautier and Vroman (2012) can be used to show that the Type-H seller's optimal reserve price is $r = s$. Letting θ_H denote the rate at which buyers visit Type-H sellers – we discuss the determination of θ_H below –, the expected payoff for a Type-H seller in competitive search equilibrium is $\Pi(s,\theta_H;s)$. Similarly, all Type-L sellers post a second-price auction with reserve price zero with corresponding expected payoff $\Pi(0,\theta_L;0)$, where θ_L, also discussed below, is the rate at which buyers visit Type-L sellers.

Given that Type-H and Type-L sellers post second-price auctions with reserve prices s and zero, respectively, buyers need to decide which seller type to visit.[6] In the *ex post* case, every buyer mixes between the two seller types with the same probability. In symmetric equilibrium, the probability, h, with which any one buyer chooses a Type-H seller has to be optimal for that buyer given that other buyers use the same mixing probability.

[6] Note that, in choosing an auction to visit, buyers care only about the reserve price. They are not concerned with the seller type *per se*. Thus, off-equilibrium beliefs are not an issue.

The arrival rates to the two seller types are determined by the buyer mixing probability. Suppose each buyer visits a randomly chosen Type-H seller with probability h. Then the arrival rate to Type-H sellers is $\frac{hB}{qS} \to (\frac{h}{q})\theta \equiv \theta_H$. The corresponding arrival rate to Type-L sellers is $\frac{(1-h)B}{(1-q)S} \to (\frac{1-h}{1-q})\theta \equiv \theta_L$. The Buyer Optimality Condition, i.e., the condition that the buyer mixing probability has to satisfy, is

$$V(0, \theta_L) \geq V(0, \theta_H) \text{ with equality if } h > 0. \tag{10}$$

Let B, L and H be the measures of buyers, Type-L sellers and Type-H sellers in the market. Taking B and L as given, we want to endogenize H. Here we are implicitly assuming that all Type-L sellers have already entered the market – if that were not the case, then no Type-H sellers would chose to enter. The free-entry condition for Type-H sellers is

$$\Pi(s, \theta_H; s) - (A + s) = 0.$$

Now consider the problem of efficient entry by Type-H sellers. The social planner chooses θ_L, θ_H and h to maximize the sum of surpluses generated by the two seller types. On a per-buyer basis, the social planner problem is thus one of choosing θ_L, θ_H and h to maximize

$$\frac{L}{B}S_L + \frac{H}{B}(S_H - (A + s)),$$

where S_L and S_H are the surpluses associated with Type-L and Type-H sellers, respectively. These are given by

$$\begin{aligned} S_L &= 1 - \int_0^1 e^{-\theta_L(1-F(x))}dx \\ S_H &= 1 - \int_s^1 e^{-\theta_H(1-F(x))}dx. \end{aligned}$$

We can rewrite this problem in terms of q and h as follows. Define $\psi = B/L$ and note that

$$\begin{aligned} \frac{H}{B} &= \frac{L}{B}\frac{H}{L} = \frac{1}{\psi}\left(\frac{q}{1-q}\right) \\ \theta_L &= (1-h)B/L = (1-h)\psi \\ \theta_H &= hB/H = h\left(\frac{1-q}{q}\right)\psi. \end{aligned}$$

The social planner problem is then one of choosing q and h to maximize

$$\Omega = \frac{1}{\psi}S_L + \frac{1}{\psi}\left(\frac{q}{1-q}\right)(S_H - (A+s)).$$

We can solve this problem by substituting for S_L and S_H and then carrying out the maximization with respect to q and h. Doing so, we can show that the equilibrium and social planner values of q and h coincide. An alternative approach is to note that since the posted mechanisms are efficient given θ_L and θ_H, we can write

$$\begin{aligned} S_L &= \Pi(0,\theta_L;0) + \theta_L V(0,\theta_L) \\ S_H &= \Pi(s,\theta_H;s) + \theta_H V(s,\theta_H). \end{aligned}$$

The first order conditions for the social planner's problem are then

$$\begin{aligned} \frac{\partial\Omega}{\partial h} &= \frac{1}{\psi}\frac{\partial S_L}{\partial\theta_L}\frac{\partial\theta_L}{\partial h} + \frac{1}{\psi}\left(\frac{q}{1-q}\right)\frac{\partial S_H}{\partial\theta_H}\frac{\partial\theta_H}{\partial h} = 0 \\ \frac{\partial\Omega}{\partial q} &= \frac{1}{\psi}\frac{\partial S_L}{\partial\theta_L}\frac{\partial\theta_L}{\partial q} + \frac{1}{\psi}\left(\frac{q}{1-q}\right)\frac{\partial S_H}{\partial\theta_H}\frac{\partial\theta_H}{\partial q} + \frac{1}{\psi}\frac{1}{(1-q)^2}(S_H - (A+s)) = 0. \end{aligned}$$

Next, we use

$$\begin{aligned} \frac{\partial\theta_L}{\partial h} &= -\psi;\ \frac{\partial\theta_H}{\partial h} = \left(\frac{1-q}{q}\right)\psi;\ \frac{\partial\theta_L}{\partial q} = 0;\ \frac{\partial\theta_H}{\partial q} = -\psi\frac{1}{q^2}h \\ \frac{\partial S_L}{\partial\theta_L} &= V(0,\theta_L);\ \frac{\partial S_H}{\partial\theta_H} = V(s,\theta_H). \end{aligned}$$

The last two equalities follow from the analogs to equation (9). Plugging all of these into the first-order conditions and using (i) Buyer Optimality, i.e., $V(0,\theta_L) = V(s,\theta_H)$, and (ii) free-entry, i.e., $\Pi(s,\theta_H;s) = A+s$, we have

Proposition 2 *In the heterogeneous-seller version of the model, when buyers draw their valuations ex post, seller entry and buyer sorting are constrained efficient.*

The intuition for the efficiency of seller entry is the same as in the homogeneous-seller case. The expected payoff for a Type-H entrant should equal her expected contribution to total surplus in the market. The entrant's auction generates an increase in market surplus equal to the expected value of the highest valuation among the buyers who participate in her auction, but she generates a decrease in market surplus equal to the expected number of buyers she steals from other auctions times the expected contribution to surplus that each of those buyers would have made elsewhere. The difference between these two terms equals the expected value of the second highest valuation drawn by buyers participating in the entrant's auction, which is the entrant's expected payoff. Buyer sorting is optimal because the incentives of individual buyers and the social planner are perfectly aligned. If the contribution of a buyer visiting, say, a Type-L seller were greater than the contribution made by a buyer at a Type-H seller, i.e., $V(0, \theta_L) > V(s, \theta_H)$, then the planner would move buyers from Type-H to Type-L sellers.

4.2 The Ex Ante Case

In the final case that we consider, buyers learn their valuations before choosing which seller to visit. As explained below, Type-L sellers again post $r = 0$ while Type-H sellers post $r = s$. Buyers then sort themselves based on their valuations. Specifically, buyers with low valuations ($x < x^*$) randomize their visits across sellers posting the zero reserve price while buyers with high valuations ($x \geq x^*$) randomize their visits across all sellers. Thus in the *ex ante* case, buyer sorting determines both expected queue lengths and the expected distributions of valuations at the two seller types.

This buyer sorting behavior can be understood as follows. It is pointless for a buyer with valuation $x \leq s$ to visit a seller posting $r = s$ since even if he were the winning bidder, his payoff would be negative. Essentially the same is true for buyers with valuations only slightly greater than s – these buyers are also better off visiting a seller posting $r = 0$, even though there is more competition there. However, why do buyers with $x \geq x^*$ randomize across *all* sellers? To understand this, consider the buyer with valuation x^*, i.e., the buyer with the lowest valuation such that he is indifferent between visiting a Type-L versus a Type-H seller. So long as buyers with higher valuations randomize their visits across all sellers, the buyer with valuation x^* is equally likely to be the high bidder in a Type-H seller's auction as he is in a Type-L seller's auction. The threshold value x^* is then determined by equating the expected payoffs in the two auctions, conditional on being the high bidder. That is, by definition, the buyer with valuation x^* is indifferent between visiting the two seller types and, thus, randomizes his visits across all sellers. The same is true for buyers with valuations above x^*. Consider a buyer with valuation $x' > x^*$. If all buyers with even higher valuations randomize their visits across all sellers, then the buyer with valuation x' is equally likely to be the high bidder in either seller type's auction. Finally, given that expected payoffs conditional on winning are the same for the buyer with valuation x^* and given that buyers with valuations between x^* and x' randomize their visits across all sellers, the expected payoff, conditional on winning, for the buyer with valuation x' is the same whether he chooses an auction with reserve price zero or one with reserve price s.

To characterize x^* explicitly, we can reason as follows. Suppose buyer x^* visits a seller posting $r = s$. Then, conditional on winning, his payoff is $x^* - s$. Suppose, alternatively, that this buyer visits a seller posting $r = 0$. Suppose n buyers with valuations below x^* visit this seller. Then, conditional on winning, buyer x^* has an expected payoff of $x^* - E[Y_n]$, where Y_n is the highest valuation drawn by these n other buyers. The density of valuations across buyers with $x < x^*$ is $f(x)/F(x^*)$; thus

$$E[Y_n] = x^* - \int_0^{x^*} \left(\frac{F(x)}{F(x^*)} \right)^n dx.$$

Buyers with valuations below x^* arrive at rate $\dfrac{BF(x^*)}{L} \to \psi F(x^*)$; hence, conditional on being the high bidder, buyer x^* can expect a payoff of

$$x^* - \sum_{n=0}^{\infty} \frac{e^{-\psi F(x^*)} \left(\psi F(x^*)\right)^n}{n!} E[Y_n] = \int_0^{x^*} e^{-\psi(F(x^*)-F(x))} dx.$$

The cutoff threshold x^*, equivalently, the Buyer Optimality Condition for the *ex ante* case, is thus characterized by

$$x^* - s = \int_0^{x^*} e^{-\psi(F(x^*)-F(x))} dx. \tag{11}$$

At the beginning of this subsection, we claimed that sellers post their reservation values. To understand why, first note that it is never in the interest of the Type-L sellers to post $r > 0$. The argument is the same as in the homogeneous-seller model. Second, given that Type-L sellers post $r = 0$, Type-H sellers will post $r = s$. There is no incentive for any Type-H seller to post a reserve price $r' > s$ since doing so would eliminate the possibility of meeting buyers with valutions between s and r' but would not affect the expected arrival rate of buyers with higher valuations. Nor does a Type-H seller want to post a reserve price below s. A Type-H seller who posted $r'' < s$ would attract additional buyers in some range (x'', x^*), but if one of these buyers were to win the auction, the seller's expected payoff would be negative. By definition, the expected value of the second highest bid given that a buyer with valuation x^* wins the auction is equal to s, so the expected value of the second highest bid given that a buyer in the range (x'', x^*) wins the auction is less than s.

We also need the free-entry condition for Type-H sellers when buyers draw their valuations *ex ante*. The free-entry condition is more complicated than the one considered in the previous subsection because in the *ex ante* case, the expected payoff for a Type-H seller depends not only on the number of buyers she can expect to attract but also on the distribution of valuations across those buyers. That is, the expected payoff for a Type-H seller can be written as $\Pi(s, \theta_H, F_H; s)$, where θ_H is now the arrival rate of buyers to Type-H sellers in the *ex ante* case and $F_H(x)$ is the distribution of valuations across buyers visiting Type-H sellers.

The free-entry condition for Type-H sellers can be written as

$$\Pi(s, \theta_H, F_H; s) = A + s.$$

Since only buyers with valuations of x^* and above visit these sellers, $\theta_H \equiv \theta(1 - F(x^*))$. We give explicit expressions for $F_H(x)$ and $\Pi(s, \theta_H, F_H; s)$ in Appendix A.2. Equation (11) and the free-entry condition describe the equilibrium in the heterogeneous-seller case in which buyers draw their valuations *ex ante*.

The final step is to consider the social planner problem. The planner chooses a level of entry by Type-H sellers and an allocation of buyer types across Type-L and Type-H sellers. If we take B and L as given (with $B/L \equiv \psi$), then, as in the *ex post* case, the level of entry by Type-H sellers, H, can be expressed in terms of $q = H/(H+L)$. The planner's allocation of buyer types across sellers reduces to a cutoff rule, $\widehat{x}$, such that all buyers with valuations below $\widehat{x}$ are directed to Type-L sellers while those with valuations of $\widehat{x}$ or above randomize their visits across all sellers. The reason the social planner uses a cutoff rule can be seen as follows. The social planner clearly wants all buyers with valuations of s and below to go to Type-L sellers. Buyers with valuations in a range from s to $\widehat{x}$ also generate more social surplus if they visit only Type-L sellers. To see this, consider a buyer with a valuation close to 1 and suppose that all the buyers with higher valuations randomize across seller types. This buyer creates social surplus only if he wins, and his probability of winning is the same at all firms. If he wins at a Type-L seller, his contribution to social surplus is his valuation minus the valuation of the next highest bidder (who would have won in his absence). If he wins at a Type-H seller, his contribution is the same unless the next highest bidder has a valuation below s, then his contribution is his valuation minus s, which is smaller. Let $\widehat{x}$ be the buyer for whom the expected next highest bidder at a Type-H seller has a valuation of s. Then the social planner wants all buyers with valuations of $\widehat{x}$ and above to randomize across all sellers. Buyers with valuations between s and $\widehat{x}$ generate more social surplus if they visit only Type-L sellers. Therefore, the planner would like to set a cutoff value so that all buyers with valuations below $\widehat{x}$ only visit the Type-L sellers and all buyers with valuations of $\widehat{x}$ and above randomize over all sellers.

As in the *ex post* case, the social planner's problem can be expressed on a per-buyer basis as

$$\Omega = \frac{1}{\psi} S_L + \frac{1}{\psi} \left(\frac{q}{1-q} \right) \left(S_H - (s + A) \right).$$

However, unlike the *ex post* case, the expected surpluses associated with Type-L and Type-H sellers depend not only on buyer arrival rates but also on the distributions of buyer valuations across the two seller types. We therefore need explicit expressions for S_L and S_H. These are derived in Appendix A.3.

The final step is to solve the social planner's optimization problem, and this is done in Appendix A.4. We show there that the social planner's cutoff value $\widehat{x}$ coincides with the privately optimal cutoff value x^* and that the free-entry equilibrium level of entry also coincides with the social optimum. That is, we show

Proposition 3 *In the heterogeneous-seller version of the model, when buyers draw their valuations ex ante, seller entry and buyer sorting are constrained efficient.*

Relative to the case in which buyers learn their valuations *ex post*, buyers are able to sort themselves across the two seller types. Proposition 3 shows that they do so optimally.

5 Conclusion

The constrained efficiency of competitive search equilibrium is well understood when meetings between buyers and sellers take place on a one-on-one basis. However, in many situations, e.g., in standard auction settings, it is more appropriate to assume a nonrival meeting technology, i.e., many-on-one meetings. These are situations in which buyers differ in terms of how much they value the good that is being offered for sale and in which these valuations are private information. In this paper, we have shown that the constrained efficiency of competitive search equilibrium continues to hold in this richer environment.

We show that competitive search equilibrium is constrained efficient in both a short-run and in a long-run sense. In the short run, i.e., taking market tightness as given, competition drives sellers to post efficient mechanisms. In a setting in which sellers post second-price auctions, this means that competition drives the symmetric equilibrium reserve price down to the common seller reservation value so that "no surplus is left on the table." We show that sellers post efficient mechanisms whether buyers learn their valuations before or after choosing which seller to visit and whether sellers are homogeneous or heterogeneous with respect to their reservation values. Our short-run efficiency results are mostly drawn from Peters and Severinov (1997) and from our earlier work (Albrecht, Gautier and Vroman 2012).

Our main contribution in this paper is to show that competitive search equilibrium is constrained efficient in the long-run sense, i.e., allowing for endogenous market tightness, even when meetings are nonrival and there is asymmetric information within meetings. To get efficiency, sellers who could potentially enter the market need the correct incentives. The payoff expected by the marginal entrant should equal her expected contribution to market surplus. If we look only at the auction that the marginal entrant posts, it seems that sellers have too little incentive to enter the market. The expected contribution to market surplus from this auction is the expectation of the highest valuation drawn by buyers who participate in that auction, while the seller's expected payoff is the expectation of the second-highest valuation across buyers in her auction. This difference between what the winning buyer expects to get and what the seller expects to receive is an "information rent," and it is the existence of this information rent that makes it seem at first glance that there will not be enough seller entry. This argument, however, neglects the fact that the buyers who participate in the marginal entrant's auction would have participated in some other seller's auction had the last seller not entered the market. That is, the marginal seller creates a business-stealing externality by her entry. Our contribution, therefore, can be understood as showing that the information-rent and business-stealing effects exactly offset each other in competitive search equilibrium, thus generating the efficient level of seller entry.

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A Appendix

A.1 Competitive search with a rival meeting technology

The Lagrangean for the seller maximization problem described in Section 3.1 is

$$L(\phi, p, \xi, \lambda) = \xi\gamma(\xi)\left(\phi + (1 - F(p))\, p\right) + \lambda \left(\gamma(\xi)(-\phi + \int_p^1 (1 - F(x))\, dx) - \overline{V}\right)$$

with first-order conditions

$$\begin{aligned}
\frac{\partial L}{\partial \phi} &= \xi\gamma(\xi) - \lambda\gamma(\xi) = 0 \\
\frac{\partial L}{\partial p} &= \xi\gamma(\xi)\left(1 - F(p) - pf(p)\right) - \lambda\gamma(\xi)(1 - F(p)) = 0 \\
\frac{\partial L}{\partial \xi} &= \left(\gamma(\xi) + \xi\gamma'(\xi)\right)\left(\phi + (1 - F(p))\, p\right) + \lambda\gamma'(\xi)(-\phi + \int_p^1 (1 - F(x))\, dx) = 0 \\
\frac{\partial L}{\partial \lambda} &= \gamma(\xi)(-\phi + \int_p^1 (1 - F(x))\, dx) - \overline{V} = 0
\end{aligned}$$

The first FOC implies that $\lambda^* = \xi$. With $\lambda^* = \xi$, the second FOC implies $p^* = 0$. Since all sellers post $p^* = 0$ and all face the same market utility constraint, $\xi^* = \theta$. Substituting into the third FOC gives

$$\phi\gamma(\theta) + \theta\gamma'(\theta) \int_0^1 (1 - F(x))\, dx = 0. \tag{12}$$

With θ fixed, the fourth FOC can be solved for $\phi^* = \phi(\theta, \overline{V})$.

With free entry, θ is endogenized. The free-entry condition when $p^* = 0$ is

$$\Pi(\phi^*, 0, \theta) = A.$$

That is,

$$\theta\gamma(\theta)\phi = A.$$

The social planner's problem is

$$\max_{\theta} \frac{\Pi(\phi^*, 0, \theta) - A}{\theta} + V(\theta)$$

or equivalently

$$\max_{\theta} \left[\frac{\theta\gamma(\theta) \int_0^1 (1 - F(x))\, dx - A}{\theta} \right].$$

The FOC for this problem is

$$\begin{aligned} \frac{\theta(\theta\gamma'(\theta) + \gamma(\theta)) \int_0^1 (1 - F(x))\, dx - (\theta\gamma(\theta) \int_0^1 (1 - F(x))\, dx - A)}{\theta^2} &= 0 \\ \theta\gamma'(\theta) \int_0^1 (1 - F(x))\, dx + \frac{A}{\theta} &= 0 \end{aligned}$$

Using equation (12), this is

$$-\phi\gamma(\theta) + \frac{A}{\theta} = 0,$$

the free-entry condition. Thus, with a rival meeting technology, a price of zero and the optimal meeting fee yield an equilibrium that is constrained efficient.

A.2 Free entry condition for the ex ante heterogeneity case

The density and cdf of valuations among those who visit Type-H sellers are

$$f_H(x) = \frac{f(x)}{1 - F(x^*)} \text{ for } x^* \leq x \leq 1 \quad (13)$$

$$F_H(x) = \frac{F(x) - F(x^*)}{1 - F(x^*)} \text{ for } x^* \leq x \leq 1. \quad (14)$$

The expected payoff for a Type-H seller is therefore

$$s + \theta(1 - F(x^*)) \int_{x^*}^1 (v(x) - s)\, e^{-\theta(1-F(x^*))(1-F_H(x))} f_H(x) dx - (x^* - s)\theta(1 - F(x^*))e^{-\theta(1-F(x^*))}$$

where the last term adjusts the virtual valuation function for the fact that when only one buyer visits the Type-H seller, the seller gets a payoff of s

rather than x^*. Using $1 - F_H(x) = \dfrac{1-F(x)}{1-F(x^*)}$, the free-entry condition for sellers in the *ex ante* case can thus be written as

$$s+\theta \int_{x^*}^{1} \left(x - \frac{1-F(x)}{f(x)} - s \right) e^{-\theta(1-F(x))} f(x)dx - (x^*-s)\theta(1-F(x^*))e^{-\theta(1-F(x^*))} = A+s. \tag{15}$$

This condition can alternatively be derived directly. The expected payoff of a Type-H seller can be written as

$$\Pi(s,\theta_H,F_H;s) = s(e^{-\theta(1-F(x^*))}+\theta(1-F(x^*))e^{-\theta(1-F(x^*))}) + \sum_{n=2}^{\infty} E[Y_{n-1}]P[N=n].$$

After much algebra, this is

$$\begin{aligned}\Pi(s,\theta_H,F_H;s) \quad &= \quad 1-(x^*-s)e^{-\theta(1-F(x^*))}(1+\theta(1-F(x^*))) \\ &\quad - \int_{x^*}^{1} e^{-\theta(1-F(x))}dx - \int_{x^*}^{1} \theta(1-F(x))e^{-\theta(1-F(x))}dx.\end{aligned}$$

which equals the left-hand side of (15).

A.3 Surpluses under ex ante heterogeneity

The expected surplus per Type-L seller in the market is computed as follows. There are $BF(x^*)$ buyers with $x < x^*$, all of whom visit Type-L sellers, and there are $B(1-F(x^*))$ buyers with $x \geq x^*$, a fraction $1-q$ of whom visit Type-L sellers (because these buyers randomize their visits across all sellers). The expected number of buyers per Type-L seller is thus $\dfrac{B\left(F(x^*)+(1-q)(1-F(x^*))\right)}{L}$, so the arrival rate of buyers to any one Type-L seller is $\psi(1-q(1-F(x^*)))$. The density of valuations across Type-L sellers is

$$f_L(x) = \begin{cases} \frac{f(x)}{1-q(1-F(x^*))} & \text{for } 0 \leq x < x^* \\ \frac{(1-q)f(x)}{1-q(1-F(x^*))} & \text{for } x^* \leq x \leq 1 \end{cases}$$

with corresponding distribution function

$$F_L(x) = \begin{cases} \frac{F(x)}{1-q(1-F(x^*))} & \text{for } 0 \leq x < x^* \\ \frac{(1-q)F(x)+qF(x^*)}{1-q(1-F(x^*))} & \text{for } x^* \leq x \leq 1 \end{cases}.$$

Conditional on receiving n visitors, the expected surplus generated in an auction held by a Type-L seller is

$$E[Y_n] = \int_0^1 xnf_L(x)F_L(x)^{n-1}dx,$$

which can be written as

$$E[Y_n] = 1 - \int_0^{x^*} \left(\frac{F(y)}{1-q(1-F(x^*))} \right)^n dy - \int_{x^*}^1 \left(\frac{(1-q)F(y)+qF(x^*)}{1-q(1-F(x^*))} \right)^n dy.$$

We can then compute S_L as follows:

$$\begin{aligned} S_L &= \sum_{n=1}^{\infty} \frac{e^{-\psi(1-q(1-F(x^*)))}\left(\psi(1-q(1-F(x^*)))\right)^n}{n!} E[Y_n] \\ &= 1 - \int_0^{x^*} e^{-\psi(1-F(y)-q(1-F(x^*)))} dy - \int_{x^*}^1 e^{-\psi(1-q)(1-F(y))} dy. \end{aligned}$$

The expected surplus per Type-H seller in the market is computed in a similar fashion. Buyers with valuations of x^* and above visit Type-H sellers at rate θ; buyers with valuations below x^* do not visit these sellers. Suppose n buyers visit a Type-H seller. The expected surplus associated with this seller is then $E\max[Y_n, s]$, where Y_n is the highest valuation among the n visitors. Since $x^* > s$, $E\max[Y_n, s] = s$ only if $n = 0$, an event that occurs with probability $e^{-\theta(1-F(x^*))}$. Using the expressions for $f_H(x)$ and $F_H(x)$ given above in Appendix A.2,

$$E[Y_n] = \int_{x^*}^1 ynf_H(x)F_H(x)^{n-1}dy = 1 - \int_{x^*}^1 \left(\frac{F(y)-F(x^*)}{1-F(x^*)} \right)^n dy, \text{ for } n = 1, 2, ..$$

Thus,

$$\begin{aligned} S_H &= se^{-\theta(1-F(x^*))} + \sum_{n=1}^{\infty} \frac{e^{-\theta(1-F(x^*))}\left(\theta(1-F(x^*))\right)^n}{n!} \left(1 - \int_{x^*}^1 \left(\frac{F(y)-F(x^*)}{1-F(x^*)} \right)^n dy \right) \\ &= 1 - (x^*-s)e^{-\theta(1-F(x^*))} - \int_{x^*}^1 e^{-\theta(1-F(y))} dy. \end{aligned}$$

Finally, using $\theta = (1-q)\psi$, we have

$$S_H = 1 - (x^*-s)e^{-(1-q)\psi(1-F(x^*))} - \int_{x^*}^1 e^{-(1-q)\psi(1-F(y))} dy.$$

A.4 Proof Proposition 3

The social planner's problem is to choose $\widehat{x}$ and q to maximize

$$\frac{1}{\psi}S_L + \frac{1}{\psi}\left(\frac{q}{1-q}\right)(S_H - (s+A))$$

where S_L and S_H are defined in Appendix A.3. Substitution gives

$$\frac{1}{\psi}\left[\begin{array}{c} 1 - \int_0^{\widehat{x}} e^{-\psi((1-F(y)-q(1-F(\widehat{x})))}dy - \int_{\widehat{x}}^1 e^{-\psi(1-q)(1-F(y))}dy \\ + \left(\frac{q}{1-q}\right)\left(1 - (\widehat{x}-s)e^{-\psi(1-q)(1-F(\widehat{x}))} - \int_{\widehat{x}}^1 e^{-\psi(1-q)(1-F(y))}dy - (s+A)\right) \end{array}\right]$$

Let

$$\Omega = \begin{array}{c} 1 - e^{\psi q(1-F(\widehat{x}))}\int_0^{\widehat{x}} e^{-\psi(1-F(y))}dy - \int_{\widehat{x}}^1 e^{-\psi(1-q)(1-F(y))}dy \\ + \left(\frac{q}{1-q}\right)\left(1 - (\widehat{x}-s)e^{-\psi(1-q)(1-F(\widehat{x}))} - \int_{\widehat{x}}^1 e^{-\psi(1-q)(1-F(y))}dy - (s+A)\right) \end{array}.$$

The first order condition with respect to $\widehat{x}$ is

$$\begin{aligned} \frac{\partial\Omega}{\partial\widehat{x}} &= [\psi q f(\widehat{x})e^{\psi q(1-F(\widehat{x}))}\int_0^{\widehat{x}} e^{-\psi(1-F(y))}dy - e^{\psi q(1-F(\widehat{x}))}e^{-\psi(1-F(\widehat{x}))} - e^{-\psi(1-q)(1-F(\widehat{x}))}] \\ &\quad + \left(\frac{q}{1-q}\right)[-e^{-\psi(1-q)(1-F(\widehat{x}))} - (\widehat{x}-s)\psi(1-q)f(\widehat{x})e^{-\psi(1-q)(1-F(\widehat{x}))} - e^{-\psi(1-q)(1-F(\widehat{x}))}] \\ &= 0 \end{aligned}$$

$$\frac{\partial\Omega}{\partial\widehat{x}} = (1-q)[\psi q f(\widehat{x})e^{\psi q(1-F(\widehat{x}))}\int_0^{\widehat{x}} e^{-\psi(1-F(y))}dy] - q(\widehat{x}-s)\psi(1-q)f(\widehat{x})e^{-\psi(1-q)(1-F(\widehat{x}))} = 0.$$

This FOC then reduces to

$$(\widehat{x}-s)e^{-\psi(1-q)(1-F(\widehat{x}))} = e^{\psi q(1-F(\widehat{x}))}\int_0^{\widehat{x}} e^{-\psi(1-F(y))}dy$$

or

$$\widehat{x} - s = \int_0^{\widehat{x}} e^{-\psi(F(\widehat{x})-F(x))}dx.$$

which implies that $\widehat{x} = x^*$.

The FOC with respect to q is

$$\begin{aligned} \frac{\partial\Omega}{\partial q} &= [-\psi(1-F(\widehat{x}))e^{\psi q(1-F(\widehat{x}))}\int_0^{\widehat{x}} e^{-\psi(1-F(y))}dy - \psi\int_{\widehat{x}}^1 e^{-\psi(1-q)(1-F(y))}(1-F(y)dy] \\ &\quad + \left(\frac{q}{1-q}\right)[-(\widehat{x}-s)\psi(1-F(\widehat{x}))e^{-\psi(1-q)(1-F(\widehat{x}))} - \psi\int_{\widehat{x}}^1 e^{-\psi(1-q)(1-F(y))}(1-F(y)dy] \\ &\quad + (\frac{1}{(1-q)^2})\left(1 - (\widehat{x}-s)e^{-\psi(1-q)(1-F(\widehat{x}))} - \int_{\widehat{x}}^1 e^{-\psi(1-q)(1-F(y))}dy - (s+A)\right) \\ &= 0 \end{aligned}$$

Substituting from the FOC for $\widehat{x}$, this simplifies to

$$
\begin{aligned}
& -\left(\frac{\psi}{1-q}\right)(\widehat{x}-s)(1-F(\widehat{x}))e^{-\psi(1-q)(1-F(\widehat{x}))}-(\frac{\psi}{1-q})\int_{\widehat{x}}^{1} e^{-\psi(1-q)(1-F(y))}(1-F(y)dy \\
= & -(\frac{1}{(1-q)^2})\left(1-(\widehat{x}-s)e^{-\psi(1-q)(1-F(\widehat{x}))}-\int_{\widehat{x}}^{1} e^{-\psi(1-q)(1-F(y))}dy-(s+A)\right).
\end{aligned}
$$

This can be rewritten as

$$
\begin{aligned}
s+A = & 1-(\widehat{x}-s)e^{-\psi(1-q)(1-F(\widehat{x}))}-\int_{\widehat{x}}^{1} e^{-\psi(1-q)(1-F(y))}dy \\
& -\psi(1-q)(\widehat{x}-s)(1-F(\widehat{x}))e^{-\psi(1-q)(1-F(\widehat{x}))} \\
& -\psi(1-q)\int_{\widehat{x}}^{1} e^{-\psi(1-q)(1-F(y))}(1-F(y)dy.
\end{aligned}
$$

Recalling that $\theta=\psi(1-q)$, this becomes

$$
\begin{aligned}
s+A = & 1-(\widehat{x}-s)e^{-\theta(1-F(\widehat{x}))}-\int_{\widehat{x}}^{1} e^{-\theta(1-F(y))}dy \\
& -\theta(\widehat{x}-s)(1-F(\widehat{x}))e^{-\theta(1-F(\widehat{x}))} \\
& -\theta\int_{\widehat{x}}^{1} e^{-\theta(1-F(y))}(1-F(y)dy.
\end{aligned}
$$

Note that this is the free entry condition.